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The Carnot engine: heat absorbed and internal energy

Guided Physics exercise · Thermodynamics

Physics Notes — Liceo Scientifico and first-year university

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November 2026

Where it comes from

This piece grew out of an interesting exchange with a fourth-year student at an Italian *liceo scientifico* (science-oriented high school), with whom I worked through a problem on the Carnot engine from his textbook: a short exercise, two questions, few calculations.

Precisely because the calculations are few, the problem is a chance to stop and think about a few fundamental concepts, which then turn out to be the key to solving it. During the solution the student asked several times why a step worked, and each question led us to clarify a point that the problem leaves implicit: what the first law of thermodynamics really says, why in an isothermal process the heat exchanged equals the work, and more.

Here those points are not buried in the calculations: each has its own space, in the Memento and Watch out boxes, so that the solution also answers the questions a problem like this one raises.

The problem

A heat engine runs on Carnot cycles, with an efficiency of 32.0% and a frequency of 10.0 Hz. During the isothermal compression, 630 J of work is done on the gas.

a) Find the heat the gas absorbs from the hot reservoir.

b) Find the change in the internal energy of the gas 10.0 s after the engine is started.

Problem 40 on p. 820 of U. Amaldi, Dalla mela di Newton al bosone di Higgs, vol. 3 — Multimediale, Zanichelli, Bologna, 2015 [1]; reworded with the same data.

Quantity	Symbol	Value
Efficiency	η	$32.0\% = 0.320$
Frequency (cycles per second)	f	10.0 Hz
Work done **on** the gas in the isothermal compression CD	$\lvert W_{CD}\rvert$	630 J
Running time	t	10.0 s
Heat absorbed from the hot reservoir	Q_2	**?**
Change in internal energy after 10.0 s	ΔU_{tot}	**?**

Conventions

- **Cycle** $ABCD$, as in the textbook: AB isothermal expansion at temperature T_2, BC adiabatic expansion, CD isothermal compression at temperature T_1, DA adiabatic compression.
- T_2 is the temperature of the **hot reservoir**, T_1 that of the **cold reservoir** (so $T_2 > T_1$).
- Q_2 is the heat absorbed from the hot reservoir, Q_1 the heat released to the cold reservoir. **Throughout this solution Q_1 and Q_2 are absolute values**, i.e. positive quantities; who absorbs and who releases is stated in words. The textbook, instead, gives heats a sign: $Q_2 > 0$ and $Q_1 < 0$. Where the textbook has Q_1, here we have its absolute value.
- In the first law $\Delta U = Q - W$, Q is the heat **absorbed** by the gas and W the work done **by** the gas, both with their sign. In a compression, work is done on the gas, so $W < 0$.

The strategy in brief

- **Section 1 — Heat absorbed.** In a Carnot cycle the gas exchanges heat only during the two isotherms. In the isothermal compression CD the internal energy does not change, so the heat released equals the work done on the gas: this gives Q_1. The efficiency then links Q_1 to Q_2.
- **Section 2 — Internal energy.** Internal energy is a state function: after each complete cycle the gas is back in its initial state. In 10.0 s the engine performs a whole number of cycles, so $\Delta U = 0$.

1. Heat absorbed from the hot reservoir

Where we are. We know the efficiency η and the work done on the gas in the isothermal compression CD. We know neither the reservoir temperatures nor the number of moles of gas.

Goal. Find Q_2, the heat absorbed from the hot reservoir in one cycle.

Idea. The efficiency links Q_2 to Q_1; in turn, Q_1 follows from the work of stage CD, because the internal energy of an ideal gas does not change along an isotherm.

Memento M1 — The Carnot cycle

The Carnot cycle consists of four **reversible** processes of an ideal gas, between two reservoirs at temperatures $T_2 > T_1$:

1. AB — **isothermal expansion** at T_2: the gas **absorbs** the heat Q_2 from the hot reservoir;
2. BC — **adiabatic expansion**: no heat exchanged, the temperature drops from T_2 to T_1;
3. CD — **isothermal compression** at T_1: the gas **releases** the heat Q_1 to the cold reservoir;
4. DA — **adiabatic compression**: no heat exchanged, the temperature rises back to T_2.

The gas exchanges heat **only during the two isotherms**: all of Q_2 enters along AB, all of Q_1 leaves along CD (Figure 1).

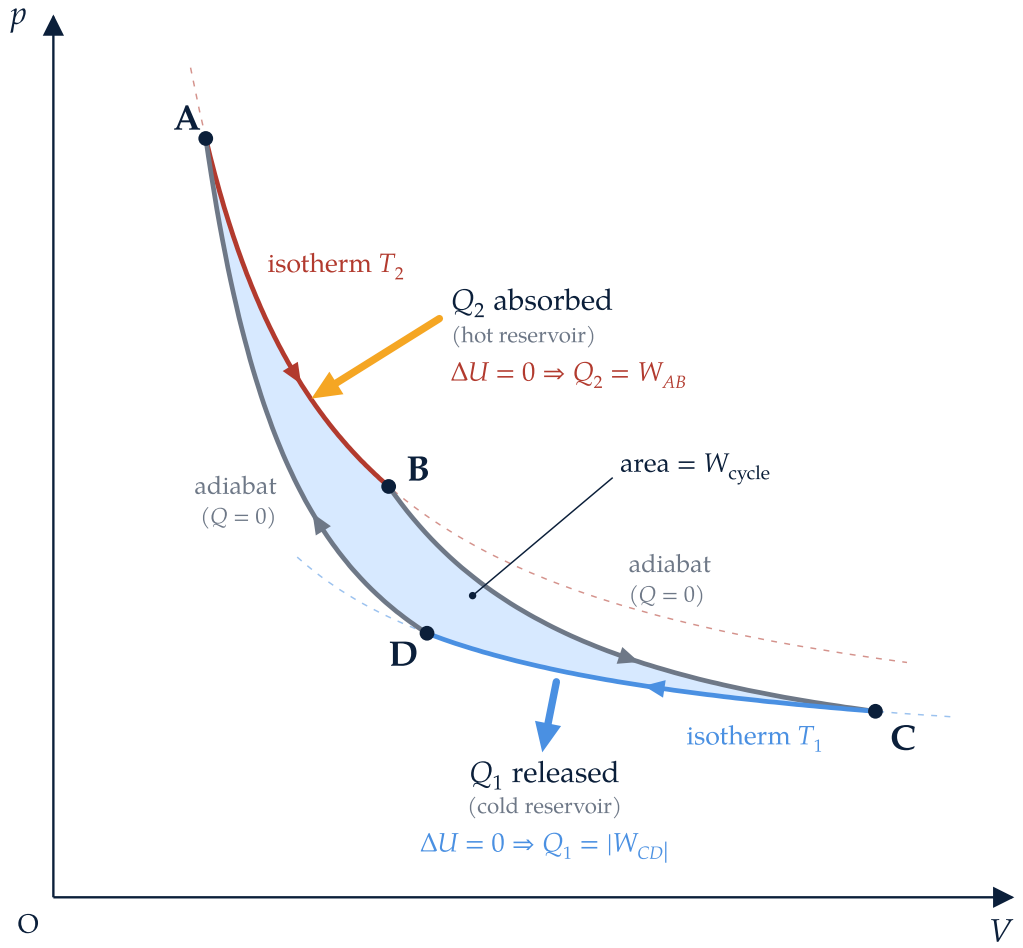

Figure 1. The Carnot cycle $ABCD$ in the p-V plane, traversed clockwise. The isotherms are arcs of hyperbolas (pV constant): in red the one at the hot-reservoir temperature T_2, in blue the one at the cold-reservoir temperature T_1; the dashed lines show their continuation. The adiabats (in grey) are steeper than the isotherms. Heat enters only along AB and leaves only along CD; along the isotherms $\Delta U = 0$, so the heat exchanged equals the work (Memento M3): $Q_2 = W_{AB}$ and $Q_1 = |W_{CD}|$. The area enclosed by the cycle is the net work W_{cycle} (Memento M4). The figure is qualitative: its proportions do not match the data of the problem.

Memento M2 — The first law of thermodynamics

The first law is **conservation of energy** applied to a thermodynamic system, such as the gas in our engine. If the gas absorbs the heat Q and does the work W, its internal energy changes by

$$\Delta U = Q - W$$

In words: the internal energy of the gas increases by the heat that flows in and decreases by the work the gas does on its surroundings. With the conventions adopted here, $Q > 0$ if the gas absorbs heat and $Q < 0$ if it releases it; $W > 0$ if the gas does work (expansion) and $W < 0$ if work is done on it (compression).

Memento M3 — Internal energy of an ideal gas

The internal energy of an ideal gas depends **only on temperature**. In an isothermal process, therefore, $\Delta U = 0$ and the first law (Memento M2) becomes

$$\Delta U = Q - W = 0 \quad \Longrightarrow \quad Q = W$$

Along an isotherm, heat and work have the same value **and the same sign**: in an expansion the gas does work ($W > 0$) and absorbs heat ($Q > 0$); in a compression work is done on it ($W < 0$) and it releases heat ($Q < 0$).

Memento M4 — Efficiency of a heat engine

The efficiency is the ratio of the work produced in one cycle to the heat absorbed from the hot reservoir. Over a complete cycle the gas returns to its initial state, so $\Delta U = 0$ and, by the first law (Memento M2), the net work is the difference between heat absorbed and heat released, $W_{\text{cycle}} = Q_2 - Q_1$ (Figure 2). With absolute values:

$$\eta = \frac{W_{\text{cycle}}}{Q_2} = \frac{Q_2 - Q_1}{Q_2} = 1 - \frac{Q_1}{Q_2}$$

For a Carnot cycle we also have $\eta = 1 - \dfrac{T_1}{T_2}$; we do not need it here, because the efficiency is given directly.

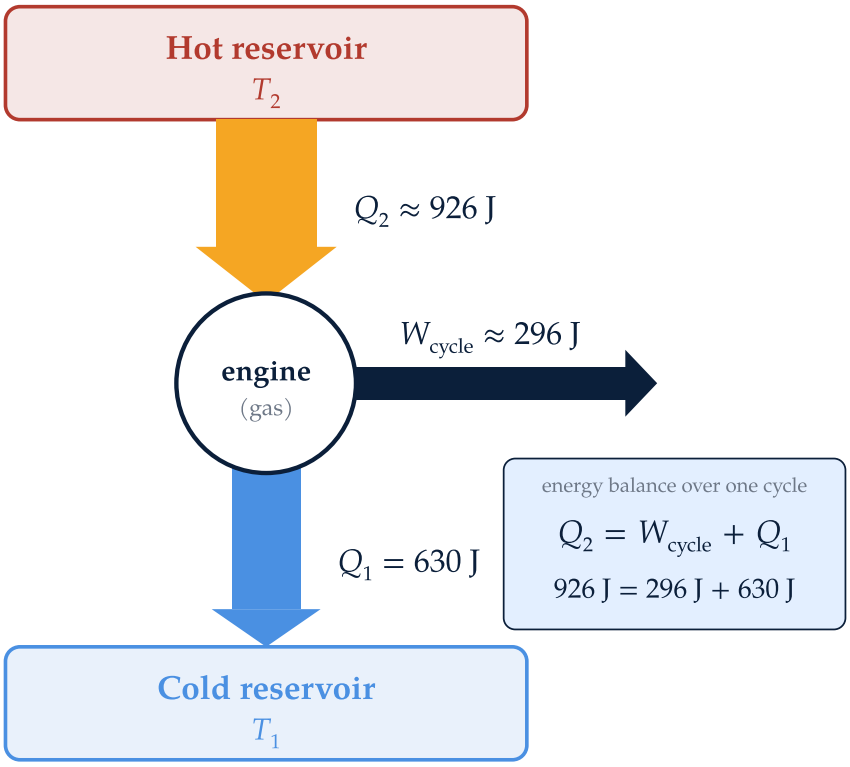

Figure 2. Diagram of a heat engine. In each cycle the gas in the engine absorbs the heat Q_2 from the hot reservoir at temperature T_2, does the work W_{cycle} and releases the heat Q_1 to the cold reservoir at temperature T_1. The width of each arrow is proportional to the energy exchanged, using the values of the problem. Since $\Delta U = 0$ over a cycle, all the energy that flows in flows out: $Q_2 = W_{\text{cycle}} + Q_1$. This is an energy balance, i.e. the conservation of energy expressed by the first law (Memento M2).

1. **Heat released to the cold reservoir.** We apply Memento M3 to the isothermal compression CD. The text says that 630 J of work is done on the gas, so the work done **by** the gas is $W_{CD} = -630$ J. Since $\Delta U_{CD} = 0$:

$$Q_{CD} = W_{CD} = -630 \text{ J}$$

The minus sign tells us the gas **releases** heat. In absolute value, all the work done on the gas leaves as heat towards the cold reservoir (Memento M1: all of Q_1 leaves along CD):

$$\boxed{Q_1 = |W_{CD}|} \tag{1}$$

$$Q_1 = 630 \text{ J}$$

2. **Heat absorbed, from the efficiency formula.** From Memento M4, $\dfrac{Q_1}{Q_2} = 1 - \eta$. We solve for Q_2 and substitute (1):

$$\boxed{Q_2 = \frac{Q_1}{1-\eta} = \frac{|W_{CD}|}{1-\eta}} \tag{2}$$

3. **Net work per cycle.** We will need it for the check and for Section 2. From Memento M4 and (2):

$$\boxed{W_{\text{cycle}} = Q_2 - Q_1 = \eta\, Q_2 = \frac{\eta}{1-\eta}\,|W_{CD}|} \tag{3}$$

4. **Discussion in terms of the efficiency.** Formulas (2) and (3) hold for any efficiency $0 < \eta < 1$. For a fixed compression work $|W_{CD}|$:

   - the factor $\dfrac{1}{1-\eta}$ is always greater than 1, so $Q_2 > Q_1$: the engine always absorbs more heat than it releases, and the difference is the work produced;
   - as $\eta \to 0$, $Q_2 \to Q_1$ and $W_{\text{cycle}} \to 0$: the engine just "passes" heat from the hot to the cold reservoir without producing work;
   - as η grows, Q_2 grows without bound: as $\eta \to 1$ the ratio Q_1/Q_2 would have to go to zero, i.e. the engine would turn almost all the absorbed heat into work. The second law of thermodynamics forbids this: for a Carnot cycle it would require $T_1/T_2 \to 0$, a cold reservoir at absolute zero.

5. **The numbers.** The efficiency goes in decimal form: $\eta = 0.320$. From (2):

$$Q_2 = \frac{630\ \text{J}}{1 - 0.320} = \frac{630\ \text{J}}{0.680} = 926.47\ldots\ \text{J} \approx 926\ \text{J}$$

   From (3):

$$W_{\text{cycle}} = Q_2 - Q_1 = 926.47\ \text{J} - 630\ \text{J} = 296.47\ \text{J} \approx 296\ \text{J}$$

   The results have three significant figures, like the data (630 J and 32.0%).

Watch out — 630 J is not the work of the cycle

A common mistake is to plug 630 J into the definition of efficiency as if it were the work of the whole cycle: $Q_2 = 630\ \text{J}/0.320 \approx 1.97 \times 10^3$ J. This is wrong: 630 J is the work of the isothermal compression CD **alone**. The net work of the cycle is a different quantity, given by (3).

Check.

- **Formula (3) and the definition of efficiency (Memento M4).**

$$\frac{W_{\text{cycle}}}{Q_2} = \frac{296.47\ \text{J}}{926.47\ \text{J}} = 0.320:$$

  we recover the 32.0% efficiency. ✓

- **Formula (2), dimensions.** The numerator is a work (joules), the denominator $1 - \eta$ is dimensionless: Q_2 comes out in joules, as a heat must. ✓
- **Formula (2), limiting case.** For $\eta = 0$, (2) gives $Q_2 = Q_1$ and (3) gives $W_{\text{cycle}} = 0$: no work produced, consistent with zero efficiency. ✓
- **Formulas (2) and (3), sign.** $Q_2 = 926\ \text{J} > Q_1 = 630\ \text{J}$, so $W_{\text{cycle}} > 0$: the engine produces work, as a heat engine should. ✓

Fun fact

At 10 cycles per second, the engine's power is $P = W_{\text{cycle}} \cdot f \approx 296\ \text{J} \times 10.0\ \text{Hz} \approx 2.96\ \text{kW}$, roughly that of a household electric oven. But beware: the processes of a Carnot cycle are reversible, i.e. ideally infinitely slow. A real engine running 10 cycles per second cannot be a Carnot engine: the numbers in the problem describe an ideal model.

Let's recap

- In a Carnot cycle the gas exchanges heat only during the isotherms (Memento M1).
- In the isothermal compression $\Delta U = 0$, so the heat released equals the work done on the gas: $Q_1 = |W_{CD}| = 630\ \text{J}$ (1).
- From the efficiency: $Q_2 = Q_1/(1 - \eta) \approx 926\ \text{J}$ (2).
- The net work per cycle is $W_{\text{cycle}} = Q_2 - Q_1 \approx 296\ \text{J}$ (3), not to be confused with the work of stage CD alone.

2. Change in internal energy after 10.0 s

Where we are. We know Q_2, Q_1 and W_{cycle} for a single cycle, and we know the engine performs 10.0 cycles per second.

Goal. Find the change in the internal energy of the gas 10.0 s after start-up.

Idea. Internal energy depends only on the state of the gas. If the gas performs a whole number of cycles in 10.0 s, it ends up back in its starting state. We assume, as is natural, that the engine starts at the beginning of a cycle.

Memento M5 — State functions

A quantity is a **state function** if its value depends only on the state of the system (pressure, volume, temperature) and not on the path by which the system got there. Internal energy U is a state function: in a cyclic process, which brings the gas back to its initial state, $\Delta U = 0$.
Heat and work, on the other hand, are **not** state functions: they depend on the path, and over a cycle they are in general not zero.

1. **Counting the cycles.** The frequency is the number of cycles per second, so in a time t:

$$\boxed{N = f\,t} \tag{4}$$

$$N = 10.0\ \text{Hz} \times 10.0\ \text{s} = 100\ \text{cycles}$$

Equivalently: each cycle lasts $1/f = 0.100$ s, and $10.0\ \text{s}/0.100\ \text{s} = 100$.

2. **Summing over complete cycles.** By Memento M5, each complete cycle gives $\Delta U_{\text{cycle}} = 0$. Over N complete cycles:

$$\boxed{\Delta U_{\text{tot}} = N\,\Delta U_{\text{cycle}} = 0} \tag{5}$$

$$\Delta U_{\text{tot}} = 100 \times 0\ \text{J} = 0\ \text{J}$$

Check.

- **Formula (5) and the first law (Memento M2).** Over one cycle the gas absorbs Q_2, releases Q_1 and does the work W_{cycle} given by (3): $\Delta U_{\text{cycle}} = (Q_2 - Q_1) - W_{\text{cycle}} = 926.47\ \text{J} - 630\ \text{J} - 296.47\ \text{J} = 0\ \text{J}$. Formula (5) is confirmed cycle by cycle. ✓
- **Formula (4), dimensions.** $\text{Hz} \cdot \text{s} = \text{s}^{-1} \cdot \text{s}$: N is dimensionless, as a number of cycles must be. ✓

Watch out — Where do the 296 J go?

In each cycle the gas absorbs 926 J and releases only 630 J, but it **does not store** the difference: the 296 J leave as work. Over 10.0 s the gas absorbs about 92.6 kJ in total, releases 63.0 kJ and does about 29.6 kJ of work: the balance is zero. Multiplying by 100 cycles makes sense for heat and work, not for ΔU.

Watch out — A whole number of cycles is needed

The result $\Delta U_{\text{tot}} = 0$ relies on $N = 100$ being a **whole number**. If the time corresponded to, say, 100.5 cycles, the gas would be somewhere in the middle of a cycle, in general at a temperature different from the initial one, and then $\Delta U \neq 0$. There is a curious exception, though: since the internal energy of an ideal gas depends only on T, we would still have $\Delta U = 0$ if the gas were anywhere on the isotherm AB, which is at the same temperature T_2 as the starting point A.

Let's recap

- Internal energy is a state function: over a complete cycle $\Delta U = 0$ (Memento M5).
- In 10.0 s the engine performs $N = f\,t = 100$ complete cycles (4).
- Hence $\Delta U_{\text{tot}} = 0$ J (5): the gas does not store energy, because it gives it back as work.

Going further — What about the temperatures?

The problem does not give the reservoir temperatures, and they are not needed. The efficiency does, however, fix their **ratio**: for a Carnot cycle $\eta = 1 - T_1/T_2$, so

$$\frac{T_1}{T_2} = 1 - \eta = 0.680$$

The individual temperatures cannot be determined: for instance, $T_2 = 500$ K would give $T_1 = 340$ K, but the pair $T_2 = 400$ K, $T_1 = 272$ K would work just as well. Likewise, the data do not allow us to find the number of moles of gas.

Historical note

The cycle is named after Sadi Carnot (1796–1832), a French military engineer who in 1824, aged 28, published a short book entitled *Réflexions sur la puissance motrice du feu* (*Reflections on the Motive Power of Fire*) [2]. Studying the steam engines of his time, Carnot realised that the efficiency of an ideal engine depends only on the temperatures of the two reservoirs. The book went almost unnoticed until, in 1834, Émile Clapeyron restated its ideas in mathematical form, introducing the p-V representation of the cycle that we still use today. It was through Clapeyron that Carnot's work reached Rudolf Clausius, who built the second law of thermodynamics upon it.

Let's recap — The full picture

Question	Result
a) Heat absorbed from the hot reservoir in one cycle	$Q_2 \approx 926$ J
b) Change in internal energy after 10.0 s	$\Delta U_{\text{tot}} = 0$ J

The results agree with those given in the textbook [1].

Bibliography

[1] U. Amaldi, *Dalla mela di Newton al bosone di Higgs*, vol. 3 — Multimediale (Quantità di moto, Dinamica rotazionale, Gravitazione, Termodinamica), Zanichelli, Bologna, 2015 (first edition 2014). ISBN 978-88-08-43636-8. Exercises of the chapter "Il secondo principio della termodinamica" (The second law of thermodynamics), problem 40, p. 820.

[2] S. Carnot, *Réflexions sur la puissance motrice du feu et sur les machines propres à développer cette puissance*, Bachelier, Paris, 1824. Analysis and digitised text (BnF/Gallica) on BibNum: https://journals.openedition.org/bibnum/858

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