keenonscience

Two stacked blocks and friction

Guided solution of a classic dynamics problem with static and kinetic friction

Physics Notes — Liceo Scientifico and first-year university

written by **Nicola Maglione** — keenonscience

keenonmaths.org · @keenonmaths

October 2026

Part 1 — The force acts on the lower block (A)

The problem

A block A of mass m_A rests on a horizontal surface, and a block B of mass m_B rests on top of it. Between A and the surface the coefficients of static and kinetic friction are μ_{As} and μ_{Ad} respectively; between B and A they are μ_{Bs} and μ_{Bd}. Air resistance is negligible. With the system initially at rest, a constant horizontal force $\vec{F}$ is applied to block A (see figure). The system is said to be in motion if at least one of the two blocks moves relative to the surface.

1. Find the value $F_{\min}$ of the magnitude of $\vec{F}$ above which the system starts to move.
2. Find the maximum value $F_{\max}$ for which B, with the system in motion, does not slip on A. Discuss, as the parameters vary, whether there are values of F for which the system moves without B slipping.
3. For the values of F for which B slips on A, find the accelerations of the two blocks relative to the surface.
4. Repeat points 1–3 assuming that $\vec{F}$ is applied to block B.
5. Numerical application. Work through the previous points with the data: (a) $m_A = 4.0$ kg, $m_B = 1.0$ kg, $\mu_{As} = 0.40$, $\mu_{Ad} = 0.30$, $\mu_{Bs} = 0.50$, $\mu_{Bd} = 0.40$; (b) $m_A = 1.0$ kg, $m_B = 4.0$ kg, $\mu_{As} = 0.20$, $\mu_{Ad} = 0.15$, $\mu_{Bs} = 0.60$, $\mu_{Bd} = 0.50$. Take $g = 9.81\ \mathrm{m/s^2}$.

In this first part we tackle points 1–3 (and the corresponding numerical application) with the force applied to A; the second part deals with the case in which the force acts on B. Whether the force pushes the block, as in the figure, or pulls it, for instance through a horizontal rope, makes no difference: the whole argument depends only on the force being horizontal and on which block receives it.

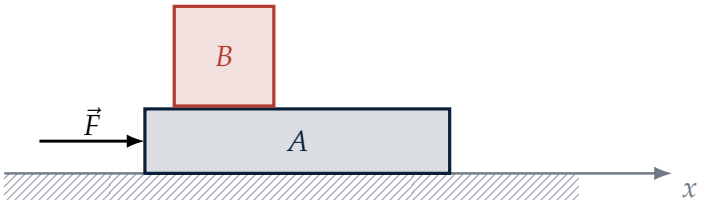

Figure 1. The situation of the problem: the force acts on the lower block.

Conventions

- The supporting surface serves as the horizontal x-axis, pointing in the direction of $\vec{F}$ (Figure 1): a force or an acceleration is positive if it points that way. For horizontal forces we write magnitudes directly, stating the direction in words or in the figure.
- All accelerations are measured relative to the surface (the ground).
- $M = m_A + m_B$ denotes the total mass of the system.
- In the friction coefficients, the first subscript indicates the contact surface (A: between A and the ground; B: between B and A), the second the type of friction: s for static, d for dynamic (often written k, for *kinetic*).
- We work symbolically until the end; numerical values are substituted only into the final formulas. We use $g = 9.81\ \mathrm{m/s^2}$.
- Numerical results are given to three significant figures (one more than the data), so that close values can be compared meaningfully.

The strategy in brief

- **Section 1 — The forces at play.** Isolate each block and draw its free-body diagram: everything else follows from here.
- **Section 2 — When the system starts moving.** As long as everything is at rest, A and B behave as a single body: we just compare F with the maximum static friction of the ground.
- **Section 3 — When B does not slip.** B is accelerated only by the friction exerted by A, which has a maximum value: this limits the common acceleration.
- **Section 4 — When B slips.** The blocks have different accelerations: Newton's second law is written separately for each, with kinetic friction between them.

- **Section 5 — Numerical application.** The symbolic results applied to the two data sets, plus a case built on purpose in which B slips right from the start.

1. The forces at play

Where we are. The system is at rest and the force $\vec{F}$ acts on A. Before computing anything, we need to know exactly which forces act on each block.

Goal. List the forces acting on A and on B and find the normal forces.

Idea. We ideally isolate each block and draw all and only the forces acting *on it* (free-body diagram). The forces the two blocks exert on each other appear in both diagrams, with opposite directions.

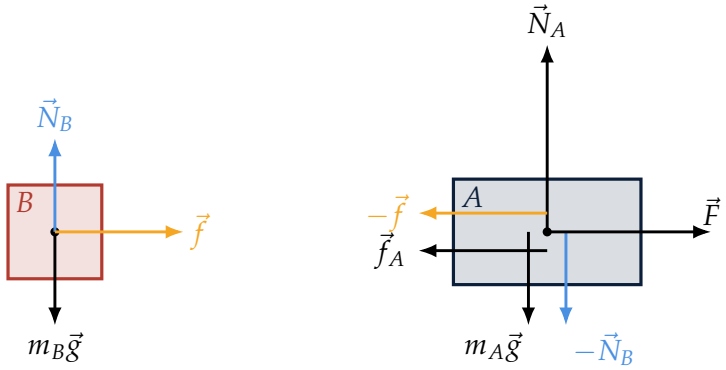

Figure 2. Free-body diagrams with the force applied to A. Forces of the same colour form action–reaction pairs.

1. **Forces on B** (Figure 2). Acting on B are its weight $m_B\vec{g}$, the normal force $\vec{N}_B$ exerted by A (upward) and the friction force $\vec{f}$ exerted by A. The direction of $\vec{f}$: when A moves (or tends to move) to the right, it "carries B along", so the friction on B points in the direction of $\vec{F}$. With the system at rest, however, we will see in Section 2 that $f = 0$. It is the only horizontal force on B: without it, B would stay put while A slides out from underneath.

2. **Forces on A.** Acting on A are its weight $m_A\vec{g}$, the force $\vec{F}$, the normal force of the ground $\vec{N}_A$ and the ground friction $\vec{f}_A$, opposite to the motion (or tendency to move) of A. In addition, by Newton's third law (Memento M1), B exerts on A a downward pressing force $-\vec{N}_B$ and a friction force $-\vec{f}$ pointing left.

3. **Normal forces.** Neither block moves vertically, so the vertical forces on each block balance. For B: $N_B = m_B g$. For A, the ground must support both the weight of A and the push of B:

$$N_A = m_A g + N_B = (m_A + m_B)\, g = Mg.$$

Memento M1 — Newton's third law

If a body A exerts a force $\vec{F}_{AB}$ on a body B, then B exerts on A a force $\vec{F}_{BA} = -\vec{F}_{AB}$: same magnitude, same line of action, opposite direction. The two forces act on **different** bodies, so they never cancel out in the diagram of a single body. The law holds for any interaction between two bodies, whether or not they belong to an isolated system, and therefore also for normal forces and friction forces.

Friction forces obey the laws recalled below: the distinction between static and kinetic friction will be at the heart of the whole exercise.

Memento M2 — Static and kinetic friction

Consider two surfaces in contact, with normal force N.

- If the surfaces **do not slide** over each other, friction is **static**: its magnitude is exactly what is needed to prevent sliding, as long as it does not exceed a maximum value:
$$f_s \leq \mu_s N.$$
- If the surfaces **slide**, friction is **kinetic** (dynamic): its magnitude is
$$f_d = \mu_d N$$
and it opposes the relative velocity of the surfaces.

Usually $\mu_d \leq \mu_s$: setting a body in motion takes more than keeping it moving.

Watch out

The normal force of the ground on A is **not** $m_A g$: the ground supports the whole system, so $N_A = (m_A + m_B)\, g$. Using $m_A g$ in the ground friction is one of the most common mistakes in this kind of exercise.

Check. Consider A and B as a single system. The forces they exchange ($\pm\vec{N}_B$ and $\pm\vec{f}$) are internal and cancel in pairs (Memento M3); vertically we are left with the total weight Mg and the normal force of the ground, which again gives $N_A = Mg$. ✓

Memento M3 — Newton's second law for a system of bodies

If several bodies move with the **same** acceleration $\vec{a}$, they can be treated as a single body whose mass is the total mass:

$$\sum \vec{F}_{\text{ext}} = M\vec{a}.$$

Only **external** forces appear in the sum: those the bodies exert on each other cancel in pairs by Newton's third law (Memento M1). The system does not need to be isolated: the system A+B is not, since the weight, $\vec{F}$ and the ground forces act on it, yet the internal forces still cancel. If instead the bodies have different accelerations, the second law must be written separately for each of them.

Historical note

The laws of sliding friction have a long history. Leonardo da Vinci, in his notebooks, observed that friction does not depend on the size of the contact area and estimated its magnitude at about a quarter of the body's weight. His notes, however, remained unpublished for centuries, and the same laws were rediscovered independently by Guillaume Amontons, who presented them to the Paris Academy of Sciences in 1699: friction is proportional to the load and independent of the apparent contact area. Charles-Augustin de Coulomb, in a memoir awarded a prize by the same Academy in 1781, systematised them and clearly distinguished friction at the onset of motion (static) from friction during sliding (kinetic). This is why the laws we use are often called the "Amontons–Coulomb laws".

Let's recap

- On B the only horizontal force is the friction $\vec{f}$ exerted by A, pointing in the direction of $\vec{F}$.
- Horizontally, A is acted on by $\vec{F}$, the ground friction $\vec{f}_A$ and the reaction $-\vec{f}$ to the friction on B.
- Normal forces: $N_B = m_B g$, $N_A = (m_A + m_B)\, g = Mg$.

2. When the system starts moving

Where we are. We know all the forces and the normal forces. The system is at rest.

Goal. Find F_{min}, the value of F above which the system starts to move (point 1).

Idea. As long as everything is at rest, A and B have the same (zero) acceleration and can be treated as a single body (Memento M3). The only external horizontal force opposing $\vec{F}$ is the ground friction, which is static and therefore has a maximum value.

1. **Equilibrium of the system.** The external horizontal forces are $\vec{F}$ and $\vec{f}_A$. For the system to stay at rest we need
$$f_A = F.$$

2. **Limit of static friction.** By Memento M2, the static friction of the ground cannot exceed $\mu_{As} N_A = \mu_{As} M g$. The system therefore stays at rest if and only if
$$F \leq \mu_{As}(m_A + m_B)\, g.$$

3. **Threshold.** It follows that
$$\boxed{F_{\min} = \mu_{As}\,(m_A + m_B)\, g} \tag{1}$$
For $F > F_{\min}$ block A starts to move.

4. **What about block B?** As long as the system is at rest, each block is in equilibrium: by Newton's first law, the net force acting on it is zero. The only horizontal force on B is the static friction $\vec{f}$ exerted by A, so with the system at rest we must have $f = 0$: the friction between the blocks is not called into play, and the whole force $\vec{F}$ is balanced by the ground. As soon as A starts moving, B must accelerate with it, and the only force that can do this is precisely the friction from A: f becomes non-zero and drags B forward. So B moves relative to the ground in any case. What remains to be seen is whether it moves *together* with A or slips on it: this is the question of point 2, and the answer is not obvious.

Check.

- Dimensions of (1): μ_{As} is dimensionless and $(m_A + m_B)\, g$ is a force, so $F_{\min}$ is a force. ✓
- Limiting case $\mu_{As} \to 0$ (perfectly smooth ground): (1) gives $F_{\min} \to 0$, i.e. any force sets the system in motion. ✓
- (1) depends on the **total** mass: to move A we must overcome the friction due to the entire weight pressing on the ground, consistently with the Watch out box of Section 1. ✓

Let's recap

- The system stays at rest as long as the static friction of the ground can balance $\vec{F}$.
- $F_{\min} = \mu_{As}(m_A + m_B)\, g$.
- Beyond the threshold A starts moving and drags B; whether B slips or not is decided in the next section.

3. When B does not slip

Where we are. For $F > F_{\min}$ the system is in motion. We ask whether B manages to follow A.

Goal. Find $F_{\max}$, the maximum value of F for which B does not slip on A, and discuss when there are values of F with the system in motion and B moving together with A (point 2).

Idea. B is accelerated **only** by the friction A exerts on it. If B does not slip, that friction is static and has a maximum value: so there is a limit to the acceleration A can "transmit" to B. We just need to compare this limit with the acceleration the system would have moving as a single block.

1. **Acceleration of B.** Suppose A and B move together with acceleration a. The only horizontal force on B is the friction $\vec{f}$ exerted by A, and it is **static** friction, because B does not slide on A. Newton's second law applied to B alone gives

   $$f = m_B\, a.$$

   Watch out: this value is **not** $\mu_{Bs} m_B g$. Static friction is "as much as needed" to prevent sliding, here exactly as much as is needed to give B the acceleration a; $\mu_{Bs} N_B$ is only its **maximum** value (Memento M2), reached in the limiting condition of step 4. Writing $f = \mu_{Bs} m_B g$ from the start would lead to wrong results.

2. **Limit set by static friction.** By Memento M2, $f \le \mu_{Bs} N_B = \mu_{Bs} m_B g$. Substituting:

   $$m_B\, a \le \mu_{Bs} m_B g \quad \Longrightarrow \quad a \le \mu_{Bs}\, g.$$

   The mass of B cancels: B does not slip if and only if the common acceleration does not exceed $\mu_{Bs} g$, **whatever the masses**.

3. **Acceleration of the system.** The system is in motion, so the ground friction is now kinetic: $f_A = \mu_{Ad} Mg$. By Memento M3:

$$F - \mu_{Ad} Mg = Ma \quad \Longrightarrow \quad a = \frac{F}{M} - \mu_{Ad}\, g.$$

4. **Condition on F.** Imposing $a \leq \mu_{Bs} g$:

$$\frac{F}{M} - \mu_{Ad}\, g \leq \mu_{Bs}\, g \quad \Longrightarrow \quad F \leq (\mu_{Ad} + \mu_{Bs})\, Mg,$$

that is

$$\boxed{F_{\max} = (\mu_{Ad} + \mu_{Bs})\,(m_A + m_B)\, g} \tag{2}$$

5. **Are there "good" values of F?** We need forces that set the system in motion ($F > F_{\min}$, formula (1)) without making B slip ($F \leq F_{\max}$, formula (2)). The interval $F_{\min} < F \leq F_{\max}$ is non-empty if and only if $\mu_{As} < \mu_{Ad} + \mu_{Bs}$, that is

$$\boxed{\mu_{As} - \mu_{Ad} < \mu_{Bs}} \tag{3}$$

6. **Interpretation.** At the instant A breaks loose, the ground friction switches from static to kinetic and "drops" abruptly from $\mu_{As} Mg$ to $\mu_{Ad} Mg$. Even with F just above the threshold, the acceleration of the system does not start from zero but from

$$a = \frac{F_{\min}}{M} - \mu_{Ad}\, g = (\mu_{As} - \mu_{Ad})\, g.$$

If this jump exceeds $\mu_{Bs}\, g$, B cannot follow A **from the very first instant**: there is no force that sets the system in motion without making B slip. A numerical example of this situation is given in Section 5.

Check.

- Dimensions of (2): $\mu_{Ad} + \mu_{Bs}$ is dimensionless and $(m_A + m_B)\, g$ is a force, so $F_{\max}$ is a force. ✓
- Limiting case $\mu_{Bs} \to 0$ (B resting on a perfectly smooth surface): (2) gives $F_{\max} = \mu_{Ad} Mg < F_{\min}$ and (3) is not satisfied, so the window does not exist and B always slips. Reasonable: without friction, nothing can drag B along. ✓
- Case $\mu_{As} = \mu_{Ad}$ (no jump between static and kinetic): (3) becomes $0 < \mu_{Bs}$, always true. The window always exists. ✓

Let's recap

- B follows A only if the common acceleration does not exceed $\mu_{Bs}g$, regardless of the masses.
- $F_{\max} = (\mu_{Ad} + \mu_{Bs})(m_A + m_B)\, g$.
- There are forces that move the system without making B slip if and only if $\mu_{As} - \mu_{Ad} < \mu_{Bs}$: the "jump" in ground friction at the start must not exceed what the friction between the blocks can withstand.

4. When B slips

Where we are. If F exceeds $F_{\max}$ (or, when the window does not exist, as soon as F exceeds $F_{\min}$), B slips on A.

Goal. Find the accelerations a_A and a_B of the two blocks (point 3).

Idea. Now the blocks have different accelerations: Memento M3 can no longer be applied to the system, and the second law must be written separately for each block, starting from new free-body diagrams. The friction between the blocks is kinetic, so its magnitude is known.

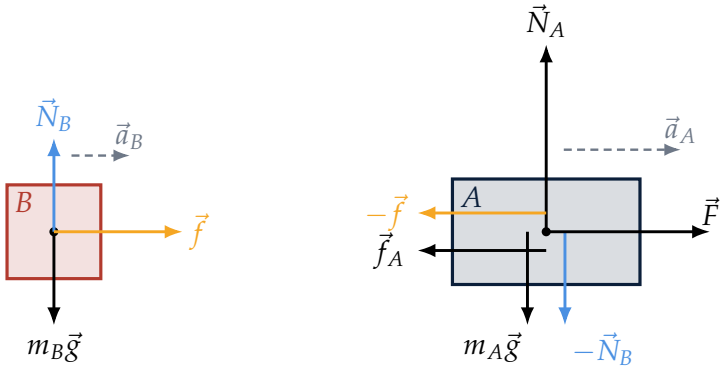

Figure 3. Free-body diagrams when B slips on A (force applied to A). The frictions are kinetic: $f = \mu_{Bd}\, m_B g$ between the blocks, $f_A = \mu_{Ad}(m_A + m_B)\, g$ with the ground. The dashed arrows are the accelerations: A accelerates more than B.

1. **What changes compared with Figure 2.** In the new diagrams (Figure 3) the forces are the same and have the same directions, but now the frictions are kinetic, so their magnitudes are known, and the two blocks have different accelerations. The direction of the friction between the blocks is the same as before: A accelerates more than B and "slides ahead" of it: relative to A, B moves backward. The kinetic friction on B opposes this relative motion, so it

points **forward** (in the direction of $\vec{F}$), with magnitude $\mu_{Bd} m_B g$. The reaction acts on A, backward (Memento M1).

2. **Block B.** The vertical forces on B balance, so $N_B = m_B g$. Horizontally, only the kinetic friction from A acts, forward, with magnitude $f = \mu_{Bd} N_B = \mu_{Bd}\, m_B g$ (Memento M2). Newton's second law along x gives:

$$m_B\, a_B = \mu_{Bd}\, m_B g \quad \Longrightarrow \quad \boxed{a_B = \mu_{Bd}\, g} \tag{4}$$

3. **Block A.** The horizontal forces on A are $\vec{F}$ forward and, backward, the reaction $-\vec{f}$ to the friction on B, with magnitude $\mu_{Bd}\, m_B g$ (Memento M1), and the kinetic friction of the ground, with magnitude $\mu_{Ad} N_A = \mu_{Ad}(m_A + m_B)\, g$. Newton's second law along x gives:

$$m_A\, a_A = F - \mu_{Bd} m_B g - \mu_{Ad}(m_A + m_B)\, g$$

$$\boxed{a_A = \frac{F - \mu_{Bd}\, m_B g - \mu_{Ad}(m_A + m_B)\, g}{m_A}} \tag{5}$$

4. **Consistency of the assumption.** We assumed that A is faster than B, i.e. $a_A > a_B$. Substituting and multiplying by m_A:

$$F - \mu_{Bd} m_B g - \mu_{Ad} M g > \mu_{Bd} m_A g \quad \Longleftrightarrow \quad F > (\mu_{Ad} + \mu_{Bd})\, M g.$$

This condition is always satisfied whenever B slips: if the window exists, by (2) $F > F_{\max} = (\mu_{Ad} + \mu_{Bs}) M g \geq (\mu_{Ad} + \mu_{Bd}) M g$ because $\mu_{Bs} \geq \mu_{Bd}$; if the window does not exist, $F > F_{\min} > F_{\max}$ and the same argument applies. ✓

Check.

- Dimensions: in (4) a_B is a pure number times g, hence an acceleration; in the numerator of (5) all terms are forces, divided by a mass. ✓
- (4) shows that a_B **does not depend on** F: once B slips, increasing F does not accelerate B any more. The only horizontal force on B is kinetic friction, whose magnitude is fixed.
- For $\mu_{Bd} \to 0$ (blocks separated by a smooth surface) (4) and (5) give $a_B = 0$ and $a_A = (F - \mu_{Ad} M g)/m_A$. B stays put while A slides underneath, still pressing on the ground with the weight of B. ✓

Fun fact

The famous trick of whipping a tablecloth out from under the dishes without knocking them over is exactly this situation: the tablecloth is A, the dishes are B. Pulling hard, the tablecloth accelerates enormously, but the dishes cannot have an acceleration greater than $\mu_{Bd}\, g$, whatever the force. If the pull is quick enough, the tablecloth is gone before the dishes have gained any appreciable speed, and they move only a few millimetres. The secret lies entirely in speed, not in force: and a smooth, hemless tablecloth and heavy dishes help a lot.

Let's recap

- When B slips, the blocks must be treated separately, with kinetic friction between them.
- $a_B = \mu_{Bd}\, g$, independent of F and of the masses.
- $a_A = \dfrac{F - \mu_{Bd} m_B g - \mu_{Ad}(m_A + m_B)\, g}{m_A}$, and it always turns out that $a_A > a_B$.

5. Numerical application

Goal. Apply the results of Sections 2–4 to the two data sets of point 5 and to a third set illustrating the case with no window.

In all the data sets of this part $M = 5.0$ kg, so $Mg = 49.05$ N.

Data (a)

$m_A = 4.0$ kg, $m_B = 1.0$ kg, $\mu_{As} = 0.40$, $\mu_{Ad} = 0.30$, $\mu_{Bs} = 0.50$, $\mu_{Bd} = 0.40$.

1. **Starting threshold**, formula (1).
$$F_{\min} = 0.40 \cdot 49.05 \text{ N} = 19.6 \text{ N}.$$
2. **Slipping threshold**, formulas (3) and (2). $\mu_{As} - \mu_{Ad} = 0.10 < \mu_{Bs} = 0.50$: the window exists.
$$F_{\max} = (0.30 + 0.50) \cdot 49.05 \text{ N} = 39.2 \text{ N}.$$
3. **A value inside the window**, $F = 30$ N:
$$a = \frac{30 \text{ N}}{5.0 \text{ kg}} - 0.30 \cdot 9.81 \text{ m/s}^2 = 3.06 \text{ m/s}^2,$$

less than $\mu_{Bs}\, g = 4.91\ \text{m/s}^2$. ✓ The friction on B is $m_B a = 3.06$ N, below the maximum of 4.91 N.

4. **A value beyond the window**, $F = 50$ N, formulas (4) and (5):

$$a_B = 0.40 \cdot 9.81\ \text{m/s}^2 = 3.92\ \text{m/s}^2;$$
$$a_A = \frac{50 - 0.40 \cdot 1.0 \cdot 9.81 - 0.30 \cdot 49.05}{4.0}\ \text{m/s}^2 = 7.84\ \text{m/s}^2.$$

✓ $a_A > a_B$.

Data (b)

$m_A = 1.0$ kg, $m_B = 4.0$ kg, $\mu_{As} = 0.20$, $\mu_{Ad} = 0.15$, $\mu_{Bs} = 0.60$, $\mu_{Bd} = 0.50$.

1. **Starting threshold**, formula (1).

$$F_{\text{min}} = 0.20 \cdot 49.05\ \text{N} = 9.81\ \text{N}.$$

2. **Slipping threshold**, formulas (3) and (2). $\mu_{As} - \mu_{Ad} = 0.05 < \mu_{Bs} = 0.60$: the window exists.

$$F_{\text{max}} = (0.15 + 0.60) \cdot 49.05\ \text{N} = 36.8\ \text{N}.$$

3. **A value inside the window**, $F = 20$ N:

$$a = \frac{20\ \text{N}}{5.0\ \text{kg}} - 0.15 \cdot 9.81\ \text{m/s}^2 = 2.53\ \text{m/s}^2 < \mu_{Bs}\, g = 5.89\ \text{m/s}^2. \quad ✓$$

4. **A value beyond the window**, $F = 50$ N, formulas (4) and (5):

$$a_B = 0.50 \cdot 9.81\ \text{m/s}^2 = 4.91\ \text{m/s}^2;$$
$$a_A = \frac{50 - 0.50 \cdot 4.0 \cdot 9.81 - 0.15 \cdot 49.05}{1.0}\ \text{m/s}^2 = 23.0\ \text{m/s}^2.$$

✓ $a_A > a_B$.

Compared with data (a), the window is lower in absolute terms (from 9.81 to 36.8 N instead of from 19.6 to 39.2 N), but much wider in relative terms: F_{max} is almost four times F_{min}, against twice for data (a). The reason is the small jump between μ_{As} and μ_{Ad} and the high μ_{Bs}.

A case with no window

We keep the masses of data (a) and the coefficients between the blocks ($\mu_{Bs} = 0.50$, $\mu_{Bd} = 0.40$), but assume a ground with $\mu_{As} = 0.80$ and $\mu_{Ad} = 0.20$. The jump between static and kinetic friction is deliberately exaggerated, to make the effect evident: it is not a typical value.

1. **Starting threshold**, formula (1).

$$F_{\min} = 0.80 \cdot 49.05\ \text{N} = 39.2\ \text{N}.$$

2. **The window does not exist**, formula (3). $\mu_{As} - \mu_{Ad} = 0.60 > \mu_{Bs} = 0.50$. Indeed, the formula for $F_{\max}$ would give

$$(0.20 + 0.50) \cdot 49.05\ \text{N} = 34.3\ \text{N},$$

   a value **less** than $F_{\min}$.

3. **Direct check**, with $F = 40$ N, just above the threshold. If the blocks moved together we would have

$$a = \frac{40\ \text{N}}{5.0\ \text{kg}} - 0.20 \cdot 9.81\ \text{m/s}^2 = 6.04\ \text{m/s}^2,$$

   greater than $\mu_{Bs}\, g = 4.91\ \text{m/s}^2$: the assumption is false, and B slips from the start.

4. **Actual accelerations**, formulas (4) and (5).

$$a_B = 0.40 \cdot 9.81\ \text{m/s}^2 = 3.92\ \text{m/s}^2;$$
$$a_A = \frac{40 - 0.40 \cdot 1.0 \cdot 9.81 - 0.20 \cdot 49.05}{4.0}\ \text{m/s}^2 = 6.57\ \text{m/s}^2.$$

   ✓ $a_A > a_B$.

Let's recap — Part 1

With the force applied to the lower block A:

	Result
Start	$F_{\min} = \mu_{As}\,(m_A + m_B)\,g$ (1)
B does not slip	$F \leq F_{\max} = (\mu_{Ad} + \mu_{Bs})(m_A + m_B)\,g$ (2)
Existence of the window	$\mu_{As} - \mu_{Ad} < \mu_{Bs}$ (3)
B slips	$a_B = \mu_{Bd}\,g$ (4), $a_A = \dfrac{F - \mu_{Bd} m_B g - \mu_{Ad}(m_A + m_B)g}{m_A}$ (5)

In the second part we will see that, by moving the force to the upper block, the structure of the problem changes completely.

Part 2 — The force acts on the upper block (B)

We return to the problem of Part 1 (same masses, same coefficients, same conventions), but now the constant horizontal force $\vec{F}$ is applied to the upper block B (point 4 of the problem). Mementos M1–M3 still apply.

The strategy in brief

- **Section 6 — The forces at play, again.** New free-body diagrams: now the friction between the blocks slows B down and drags A along.
- **Section 7 — When the system starts moving, again.** The system can start in two ways, with B sliding on A or with A sliding on the ground: the threshold is the lower of the two, and this distinguishes two regimes.
- **Section 8 — When B does not slip, again.** In the first regime the question has no answer; in the second we compute the friction B needs to drag A along.
- **Section 9 — When B slips, again.** For B the calculation is immediate; for A we must first see whether the friction it receives is enough to break it loose from the ground.
- **Section 10 — Numerical application.** The two data sets, which here fall into opposite regimes.

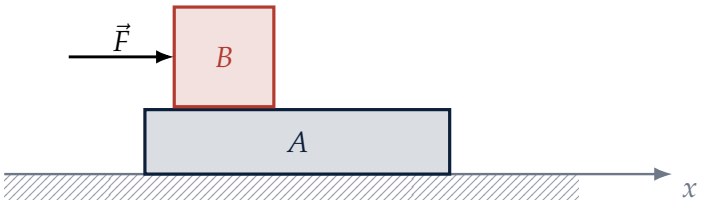

Figure 4. The force now acts on the upper block.

6. The forces at play, again

Where we are. The situation is that of Part 1, but $\vec{F}$ acts on B.

Goal. Redraw the free-body diagrams and understand what changes.

Idea. The vertical forces do not change ($\vec{F}$ is horizontal); what changes are the directions of the friction forces between the blocks, because now it is B that "drags" A and not the other way round.

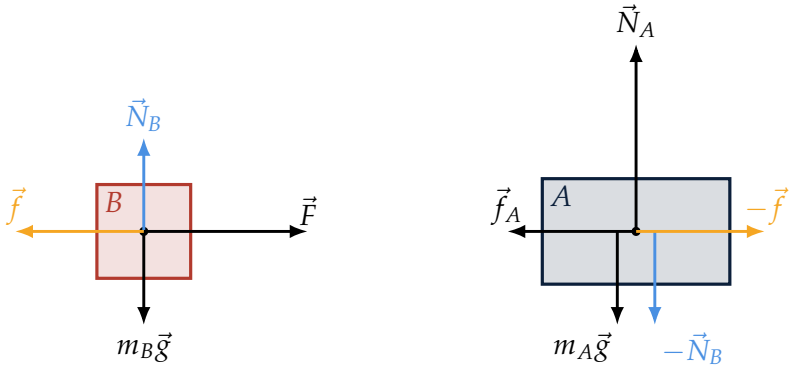

Figure 5. Free-body diagrams with the force applied to B. Now it is the friction from B that drags A forward.

1. **Forces on B** (Figure 5). Weight $m_B\vec{g}$, normal force $\vec{N}_B$, the force $\vec{F}$ forward and the friction $\vec{f}$ exerted by A, which opposes B's tendency to slide forward: so it points **backward**.

2. **Forces on A.** Weight, ground normal force $\vec{N}_A$, pressing force $-\vec{N}_B$ and ground friction $\vec{f}_A$ backward. By Newton's third law (Memento M1), the friction from B on A points **forward**: it is now the **only** horizontal force that can set A in motion.

3. **Normal forces.** Unchanged: $N_B = m_B g$ and $N_A = (m_A + m_B)\, g = Mg$.

Check. On the system A+B the external horizontal forces are still only $\vec{F}$ and $\vec{f}_A$: from the point of view of the system as a whole, applying the force to A or to B makes no difference. The whole difference lies in how the force is transmitted between the blocks. ✓

Let's recap

- Normal forces unchanged: $N_B = m_B g$, $N_A = Mg$.
- The friction between the blocks slows B down and drags A along: the directions are opposite to those of Part 1.
- A can move only thanks to the friction from B, whose magnitude is limited.

7. When the system starts moving, again

Where we are. The system is at rest and $\vec{F}$ acts on B.

Goal. Find F_{min} (point 4, repeating point 1).

Idea. Now the system can start moving in two ways: B slides on A, or A (with B on top) slides on the ground. The system stays at rest only if **both** static frictions hold: the threshold is the lower of the two.

1. **Equilibrium of B.** If B is at rest, by Newton's first law the net horizontal force on B is zero: the static friction from A must exactly balance $\vec{F}$, i.e. $f = F$. By Memento M2 we need $F \leq \mu_{Bs} N_B = \mu_{Bs} m_B g$.
2. **Equilibrium of the system.** If everything is at rest, the net external force on the system A+B is also zero (Memento M3, with zero acceleration): the static friction of the ground balances $\vec{F}$, i.e. $f_A = F$. We need $F \leq \mu_{As} Mg$.
3. **Threshold.** Everything stays at rest if and only if both conditions hold, so

$$\boxed{F_{\text{min}} = \min\{\mu_{Bs}\, m_B,\ \mu_{As}(m_A + m_B)\}\, g} \tag{6}$$

4. **Who moves first: two regimes.**
   - **Regime (i):** $\mu_{Bs} m_B \leq \mu_{As}(m_A + m_B)$**.** The lower threshold is the one between the blocks: $F_{\text{min}} = \mu_{Bs} m_B g$, and it is **B** that starts moving, sliding on A. Block A stays at rest: from B it receives at most the kinetic friction $\mu_{Bd} m_B g$, which does not exceed $\mu_{Bs} m_B g \leq \mu_{As} Mg$, and so is not enough to overcome the static friction of the ground.
   - **Regime (ii):** $\mu_{Bs} m_B > \mu_{As}(m_A + m_B)$**.** The lower threshold is that of the ground: $F_{\text{min}} = \mu_{As} Mg$, and it is **A** that starts moving, with B on top.

5. **When are we in regime (ii)?** Let us rewrite the condition as

$$m_B\,(\mu_{Bs} - \mu_{As}) > \mu_{As}\,m_A.$$

The right-hand side is positive, so we need at least $\mu_{Bs} > \mu_{As}$ (B grips A better than A grips the ground) and, in addition, B must be heavy enough compared with A.

Check.

- Dimensions of (6): both terms of the minimum are a mass times a dimensionless coefficient; multiplied by g they give a force. ✓
- Limiting case $m_A \to 0$ with $\mu_{Bs} > \mu_{As}$: the regime (ii) condition becomes $\mu_{Bs} > \mu_{As}$, true. Reasonable: if A has no mass, it is as if B rested directly on the ground with a "mat" in between, and it slides where friction is lower. ✓

Let's recap

- $F_{\min} = \min\{\mu_{Bs}\,m_B,\ \mu_{As}(m_A + m_B)\}\,g$.
- Regime (i): B starts, sliding on A; A stays at rest.
- Regime (ii), possible only if $\mu_{Bs} > \mu_{As}$ and B is heavy enough: A starts, with B on top.

8. When B does not slip, again

Where we are. We know the starting threshold and the two regimes.

Goal. Find $F_{\max}$, if it exists (point 4, repeating point 2).

Idea. In regime (i) the answer is immediate: the system starts moving precisely because B slips. In regime (ii) we reason as in Section 3: we assume the blocks move together and compute how much static friction B needs.

1. **Regime (i).** The system moves only when B slips on A: there is **no** value of F for which the system moves without B slipping. Point 2 has no answer.
2. **Regime (ii): common acceleration.** For $F > F_{\min}$ the blocks start together (we check this in step 5). The ground friction is kinetic, and by Memento M3

$$a = \frac{F}{M} - \mu_{Ad}\,g.$$

3. **Friction needed on B.** Horizontally, B is acted on by $\vec{F}$ forward and by the friction $\vec{f}$ exerted by A, backward; it is **static** friction, because B does not slide on A. As in Section 3, f is as much as is needed to give B the common acceleration, not $\mu_{Bs} m_B g$: the maximum value comes into play only in the limiting condition of step 4. Newton's second law for B gives $F - f = m_B a$, so

$$f = F - m_B a = F - m_B \left(\frac{F}{M} - \mu_{Ad}\, g \right) = F\, \frac{m_A}{M} + \mu_{Ad}\, m_B g.$$

   The friction needed increases with F.

4. **No-slip condition.** We impose $f \leq \mu_{Bs} m_B g$:

$$F\, \frac{m_A}{M} \leq (\mu_{Bs} - \mu_{Ad})\, m_B g \quad \Longrightarrow$$

$$\boxed{F_{\max} = \frac{m_B\,(m_A + m_B)}{m_A}\,(\mu_{Bs} - \mu_{Ad})\, g} \tag{7}$$

5. **Consistency: the window always exists, and B does not slip at the start.** Since f increases with F, it is enough to check that at the starting threshold the friction needed is below the maximum. For F just above $F_{\min} = \mu_{As} M g$ (formula (6), regime (ii)):

$$f = \mu_{As} M g\, \frac{m_A}{M} + \mu_{Ad}\, m_B g = (\mu_{As}\, m_A + \mu_{Ad}\, m_B)\, g.$$

   The regime (ii) condition, $\mu_{Bs} m_B > \mu_{As} m_A + \mu_{As} m_B$, together with $\mu_{As} \geq \mu_{Ad}$, gives

$$\mu_{Bs}\, m_B > \mu_{As}\, m_A + \mu_{Ad}\, m_B,$$

   that is $f < \mu_{Bs} m_B g$: at the start B follows A, and consequently $F_{\max} > F_{\min}$. ✓

6. **Comparison with Part 1.** There, the "jump" in ground friction at the start could make B slip immediately; here it cannot. The reason: when the force acts on B, the friction between the blocks only has to accelerate A, and the acceleration at the start is largely "paid for" directly by $\vec{F}$.

Check.

- Dimensions of (7): $\dfrac{m_B\,(m_A + m_B)}{m_A}$ is a mass, multiplied by a dimensionless coefficient and by g. ✓
- Limiting case $m_A \to 0$: (7) gives $F_{\max} \to \infty$. A massless block A needs no force to be accelerated, so B never slips. ✓
- In (7) $F_{\max}$ depends on μ_{Ad} with a minus sign: a "rougher" ground during motion makes slipping easier, because A offers more resistance and B has to "drag" it harder. ✓

Let's recap

- Regime (i): $F_{\max}$ does not exist; the system moves only with B slipping.
- Regime (ii): $F_{\max} = \dfrac{m_B\,(m_A + m_B)}{m_A}\,(\mu_{Bs} - \mu_{Ad})\,g$, always greater than $F_{\min}$.
- Unlike Part 1, B never slips at the start: the window always exists in regime (ii).

9. When B slips, again

Where we are. B slips on A: in regime (i) for $F > F_{\min}$, in regime (ii) for $F > F_{\max}$.

Goal. Find a_A and a_B (point 4, repeating point 3).

Idea. As in Section 4, we write the second law separately for the two blocks, with kinetic friction between them. For B it is immediate. For A there is an extra question: is the friction it receives from B enough to set it in motion?

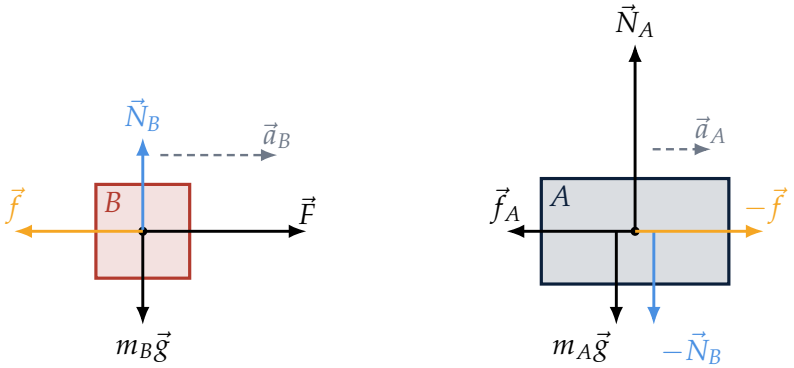

Figure 6. Free-body diagrams when B slips on A (force applied to B, sub-case (ii-a)). The frictions are kinetic: $f = \mu_{Bd}\,m_B g$ between the blocks, $f_A = \mu_{Ad}(m_A + m_B)\,g$ with the ground. The dashed arrows are the accelerations: B accelerates more than A. If instead A stays at rest, $\vec{a}_A = \vec{0}$ and $\vec{f}_A$ is a static friction that balances $-\vec{f}$.

1. **Block B** (Figure 6). The vertical forces on B balance, so $N_B = m_B g$. Horizontally, $\vec{F}$ acts forward and the kinetic friction from A acts backward, with magnitude $f = \mu_{Bd} N_B = \mu_{Bd}\,m_B g$ (Memento M2). Newton's second law along x gives:

$$m_B\,a_B = F - \mu_{Bd}\,m_B g \quad\Longrightarrow\quad \boxed{a_B = \frac{F}{m_B} - \mu_{Bd}\,g} \tag{8}$$

2. **Block A: does it start or not?** The only force pushing A forward is the kinetic friction from B, of magnitude $\mu_{Bd} m_B g$, **independent of** F. A is initially at rest, so this force must be compared with the maximum **static** friction of the ground, $\mu_{As} M g$.

   - **If** $\mu_{Bd}\, m_B \leq \mu_{As}(m_A + m_B)$**:** A stays at rest, $a_A = 0$. This always happens in regime (i) (Section 7, step 4) and can also happen in regime (ii): we call this sub-case (ii-b). It requires a large jump between μ_{Bs} and μ_{Bd}.
   - **If** $\mu_{Bd}\, m_B > \mu_{As}(m_A + m_B)$ (sub-case (ii-a)): A starts moving, and now the ground friction is kinetic, with magnitude $\mu_{Ad} N_A = \mu_{Ad}(m_A + m_B)\, g$, pointing backward. Newton's second law along x for A gives:

$$m_A\, a_A = \mu_{Bd}\, m_B g - \mu_{Ad}(m_A + m_B)\, g$$

$$\boxed{a_A = \frac{\mu_{Bd}\, m_B - \mu_{Ad}(m_A + m_B)}{m_A}\, g} \tag{9}$$

3. **Consistency in sub-case (ii-a).** We assumed that B slides forward relative to A, i.e. $a_B > a_A$. Multiplying by $m_A m_B$ and simplifying, the condition becomes

$$F > \frac{m_B\,(m_A + m_B)}{m_A}\,(\mu_{Bd} - \mu_{Ad})\, g,$$

   always satisfied because the right-hand side does not exceed $F_{\max}$ from (7) (just compare them: $\mu_{Bd} \leq \mu_{Bs}$). ✓

Watch out

To decide whether A starts, the friction received from B must be compared with the **static** friction of the ground (μ_{As}), not the kinetic one: A is at rest, and must first "break loose". Only afterwards, to compute its acceleration, do we use μ_{Ad}. Mixing up the two steps leads to setting A in motion when in fact it stays at rest.

Check.

- Dimensions: in (8) F/m_B is an acceleration, like $\mu_{Bd}\, g$; in (9) the ratio of masses is dimensionless and multiplies g. ✓
- (9) shows that, in sub-case (ii-a), a_A **does not depend on** F: this is the counterpart of what we saw in Part 1 for a_B. The block that does not receive the force directly is moved only by kinetic friction, whose magnitude is fixed.
- In sub-case (ii-a) (9) gives $a_A > 0$, since $\mu_{Bd} m_B > \mu_{As} M \geq \mu_{Ad} M$. ✓

Let's recap

- $a_B = \dfrac{F}{m_B} - \mu_{Bd}\, g$.
- A starts only if $\mu_{Bd} m_B > \mu_{As}(m_A + m_B)$, in which case $a_A = \dfrac{\mu_{Bd} m_B - \mu_{Ad}(m_A + m_B)}{m_A}\, g$, independent of F; otherwise $a_A = 0$.
- The block that does not receive the force directly is moved only by kinetic friction: its acceleration does not depend on F.

10. Numerical application

Goal. Apply the results of Sections 7–9 to the two data sets. Here too $Mg = 49.05$ N.

Data (a)

$m_A = 4.0$ kg, $m_B = 1.0$ kg, $\mu_{As} = 0.40$, $\mu_{Ad} = 0.30$, $\mu_{Bs} = 0.50$, $\mu_{Bd} = 0.40$.

1. **Regime.** $\mu_{Bs} m_B = 0.50$ kg, $\mu_{As} M = 2.0$ kg: we are in regime (i).
2. **Starting threshold**, formula (6).

   $$F_{\min} = 0.50 \cdot 1.0 \cdot 9.81 \text{ N} = 4.91 \text{ N}:$$

   B starts.
3. **Slipping threshold.** It does not exist: every force that moves the system makes B slip.
4. **Accelerations**, with $F = 10$ N, formula (8):

   $$a_B = \frac{10}{1.0}\ \text{m/s}^2 - 0.40 \cdot 9.81\ \text{m/s}^2 = 6.08\ \text{m/s}^2;$$

   $a_A = 0$: A receives from B only

   $$0.40 \cdot 1.0 \cdot 9.81 \text{ N} = 3.92 \text{ N},$$

   far less than the 19.6 N needed to break it loose from the ground.

Data (b)

$m_A = 1.0$ kg, $m_B = 4.0$ kg, $\mu_{As} = 0.20$, $\mu_{Ad} = 0.15$, $\mu_{Bs} = 0.60$, $\mu_{Bd} = 0.50$.

1. **Regime.** $\mu_{Bs} m_B = 2.4\ \text{kg} > \mu_{As} M = 1.0\ \text{kg}$: regime (ii). Moreover $\mu_{Bd} m_B = 2.0\ \text{kg} > 1.0\ \text{kg}$: sub-case (ii-a).

2. **Starting threshold**, formula (6).

$$F_{\min} = 0.20 \cdot 49.05\ \text{N} = 9.81\ \text{N}:$$

   they start together.

3. **Slipping threshold**, formula (7).

$$F_{\max} = \frac{4.0 \cdot 5.0}{1.0} \cdot (0.60 - 0.15) \cdot 9.81\ \text{N} = 88.3\ \text{N}.$$

4. **A value inside the window**, $F = 50\ \text{N}$:

$$a = \frac{50\ \text{N}}{5.0\ \text{kg}} - 0.15 \cdot 9.81\ \text{m/s}^2 = 8.53\ \text{m/s}^2.$$

   The friction needed on B is

$$f = 50\ \text{N} \cdot \frac{1.0}{5.0} + 0.15 \cdot 4.0 \cdot 9.81\ \text{N} = 15.9\ \text{N},$$

   below the maximum $\mu_{Bs} m_B g = 23.5\ \text{N}$. ✓

5. **A value beyond the window**, $F = 100\ \text{N}$, formulas (8) and (9):

$$a_B = \frac{100}{4.0}\ \text{m/s}^2 - 0.50 \cdot 9.81\ \text{m/s}^2 = 20.1\ \text{m/s}^2;$$
$$a_A = \frac{0.50 \cdot 4.0 - 0.15 \cdot 5.0}{1.0} \cdot 9.81\ \text{m/s}^2 = 12.3\ \text{m/s}^2.$$

   ✓ $a_B > a_A$.

The same data that in Part 1 gave an ordinary window (data (a)) here remove it entirely; conversely, data (b), with B heavy and gripping A well, produce a huge window, from 9.81 to 88.3 N.

Let's recap — The full picture

	$\vec{F}$ on A	$\vec{F}$ on B
Start	$F_{\min} = \mu_{As}\, Mg$ (1)	$F_{\min} = \min\{\mu_{Bs} m_B,\ \mu_{As} M\}\, g$ (6)
B does not slip	$F_{\max} = (\mu_{Ad} + \mu_{Bs})\, Mg$ (2)	only in regime (ii): $F_{\max} = \dfrac{m_B M}{m_A}(\mu_{Bs} - \mu_{Ad})\, g$ (7)
Existence of the window	$\mu_{As} - \mu_{Ad} < \mu_{Bs}$ (3)	$\mu_{Bs} m_B > \mu_{As} M$
B slips: block receiving $\vec{F}$	$a_A = \dfrac{F - \mu_{Bd} m_B g - \mu_{Ad} Mg}{m_A}$ (5)	$a_B = \dfrac{F}{m_B} - \mu_{Bd}\, g$ (8)
B slips: dragged block	$a_B = \mu_{Bd}\, g$ (4)	$a_A = \dfrac{\mu_{Bd} m_B - \mu_{Ad} M}{m_A}\, g$ (9) if $\mu_{Bd} m_B > \mu_{As} M$, otherwise 0

($M = m_A + m_B$.) Two ideas run through the whole exercise. The first: static friction is "as much as needed" and has only a maximum value, whereas kinetic friction has a fixed value. The second: the block that does not receive the force directly is moved only by friction, so its acceleration has a limit that no force can exceed.

About the author. Nicola Maglione is an electronic engineer (degree in Electronic Engineering, University of Naples, 2001), with over twenty years of experience in the semiconductor industry as a design and program manager, specialized in NAND Flash memory and high-speed interfaces, and holder of several patents in memory architectures. Alongside his professional work, he has always cultivated a passion for mathematics — particularly Analysis, Abstract Algebra, and Number Theory — and for physics, which he shares through keenonscience, his science-outreach page, and on the website keenonmaths.org.

© 2026 Nicola Maglione. Distributed under a Creative Commons Attribution — NonCommercial — NoDerivatives 4.0 International license (CC BY-NC-ND 4.0): sharing is permitted with attribution; commercial use and derivative works are not permitted.
creativecommons.org/licenses/by-nc-nd/4.0