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Parametric inequalities: which values should you discuss?

How to tell which values of the parameter to discuss

Mathematics Notes — Science High School

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The question

It all started with a question asked at the beginning of a lesson by a second-year student at an Italian *liceo scientifico* (science-oriented high school):

> *"I know how to solve inequalities. But with parametric ones I don't understand how to do the discussion: how do I know which values of the parameter I have to discuss?"*

We could hand out a rule to memorise straight away, but it wouldn't help much. Instead, we will solve a parametric inequality from start to finish, watching **at what moment** and **for what reason** each value of the parameter that needs discussing shows up. The answer to the question will emerge by itself, and we will then put it to the test on a second exercise.

The strategy in brief

- **Part I — First exercise (sections 1–5).** We solve a rational inequality with a parameter, studying the sign of numerator and denominator separately.
  - Section 1: domain conditions and setting up the sign study.
  - Section 2: the numerator. To isolate x we must divide by an expression containing a, and here the **first critical value** appears.
  - Sections 3–4: to build the sign chart we must know the order of the points on the x-axis. Comparing them, the **second critical value** appears.
  - Section 5: summary and check.
- **Part II — The answer (section 6).** We look back over the exercise and extract the three "sources" the values to discuss come from, together with a practical procedure.
- **Part III — The direct method (sections 7–9).** With a second exercise we put the answer to the test and, at the same time, use it as a method: **first** we find all the critical values, **then** we solve. It is the quickest and most efficient approach, the one to use in a class test.
- **Appendix A (optional).** The first exercise seen all at once, in a single figure in the (a, x) plane.

Part I — First exercise

Discuss, as the parameter $a \in \mathbb{R}$ varies, the inequality

$$\frac{(a-2)x - 3a}{x-2} > 0 \tag{1}$$

1. Domain conditions and setup

Where we are. We have a rational inequality in which, besides the unknown x, a real parameter a appears.

Goal. Establish where (1) is meaningful and set up the study of the sign of the fraction.

Idea. The sign of a fraction depends on the signs of numerator and denominator: we study them separately and then combine them in a sign chart (Memento M1).

1. **Domain conditions.** The denominator must not vanish:

$$x - 2 \neq 0 \iff x \neq 2.$$

   This condition **does not depend on** a: for every $a \in \mathbb{R}$, (1) is meaningful for $x \in \mathbb{R} \setminus \{2\}$.

2. **Numerator and denominator.** Let

$$N(x) = (a-2)x - 3a, \qquad D(x) = x - 2.$$

   We will study $N(x) \geq 0$ and $D(x) \geq 0$.

3. **The denominator** is the simpler one and does not contain the parameter:

$$\boxed{D(x) \geq 0 \iff x \geq 2} \tag{2}$$

Memento M1 — Sign of a fraction

A fraction $\dfrac{N}{D}$ is positive where N and D have the same sign, negative where they have opposite signs, zero where $N = 0$ (and $D \neq 0$). Where $D = 0$ it is undefined. In the sign chart the sign of N and the sign of D go on separate rows, and they are combined column by column with the rule of signs.

Watch out — Why $D(x) \geq 0$, and why that point is never a solution

We write $D(x) \geq 0$ to treat numerator and denominator in the same way: this makes the point where the denominator vanishes appear in the sign chart, which is exactly what we need to build it. But at that point the fraction **does not exist**: it can never be part of the solution, not even when the inequality is of the ≥ 0 type. As a reminder, in the chart we mark it with a red ×. Some textbooks and teachers prefer to write $D(x) > 0$ precisely to avoid this misunderstanding: the two approaches lead to the same result, as long as the excluded point is left out.

Let's recap

- Domain conditions: $x \neq 2$, for every value of a.
- The sign of the denominator, formula (2), does not depend on a: positive for $x > 2$, zero at $x = 2$, where the fraction does not exist.
- What remains is the numerator, which is where the parameter comes into play.

2. The numerator: the first critical value

Where we are. We know the domain conditions and the sign of the denominator, formula (2).

Goal. Solve $N(x) \geq 0$.

Idea. $N(x) \geq 0$ is a linear inequality in x: to isolate x we must divide by the coefficient $a - 2$. But we don't know the sign of $a - 2$, and it could even be zero.

1. Move the constant term to the right:

$$N(x) \geq 0 \iff (a-2)x \geq 3a.$$

2. To divide by $a - 2$ we need to know whether it is positive, negative or zero (Memento M2). We are forced to distinguish three cases.

   - **If** $a > 2$, the coefficient is positive and the inequality keeps its direction:

$$x \geq \frac{3a}{a-2} \tag{3}$$

   - **If** $a < 2$, the coefficient is negative and the inequality reverses:

$$x \leq \frac{3a}{a-2} \tag{4}$$

- **If** $a = 2$, the coefficient is zero and we cannot divide. We substitute $a = 2$ directly into the original inequality (1):

$$\frac{-6}{x-2} > 0 \iff x - 2 < 0 \iff x < 2.$$

Hence

$$\boxed{a = 2 : \quad x < 2} \tag{5}$$

3. For convenience, when $a \neq 2$ we call

$$x_N = \frac{3a}{a-2}$$

the value of x at which the numerator vanishes. For $a = 2$ this expression does not exist: one more confirmation that $a = 2$ had to be treated separately.

Memento M2 — Dividing both sides of an inequality

Multiplying or dividing both sides by a **positive** quantity keeps the direction; by a **negative** quantity it reverses the direction. You cannot divide by **zero**.

Watch out — Never divide by something that can be zero

When the coefficient of x contains the parameter, the values of a that make it zero must be treated separately, by substituting them into the **original** inequality, not into the ones derived later.

Check. Let us test (5) on (1) with $a = 2$. For $x = 0$ the fraction equals $\frac{-6}{-2} = 3 > 0$: (1) holds, and indeed $0 < 2$. For $x = 3$ it equals $\frac{-6}{1} = -6 < 0$: it does not hold, and indeed $3 > 2$.

Let's recap

- **First critical value:** $a = 2$. It arises because the coefficient of x in the numerator, $a - 2$, vanishes for that value and prevents us from dividing.
- For $a = 2$ the solution is $x < 2$, formula (5).
- The cases $a > 2$, formula (3), and $a < 2$, formula (4), still need to be completed.

3. The case $a > 2$

Where we are. For $a > 2$ we know that $N(x) \geq 0 \iff x \geq x_N$, formula (3), and that $D(x) \geq 0 \iff x \geq 2$, formula (2).

Goal. Build the sign chart and find the solution.

Idea. To draw the chart we must know whether, on the x-axis, the point x_N lies to the right or to the left of 2. We compare them by studying the sign of their difference (Memento M3).

1. Compute the difference:

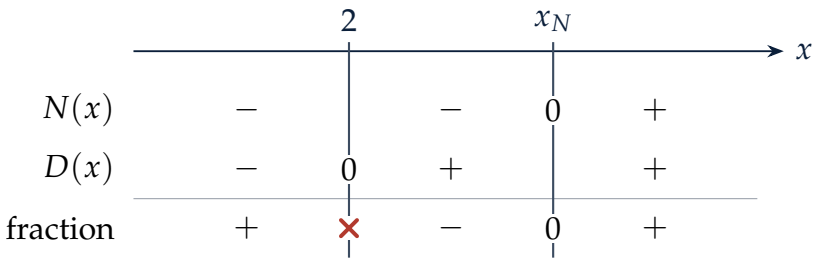
$$x_N - 2 = \frac{3a}{a-2} - 2 = \frac{3a - 2a + 4}{a-2},$$

that is

$$\boxed{x_N - 2 = \frac{a+4}{a-2}} \tag{6}$$

2. For $a > 2$ both numerator and denominator of (6) are positive, so $x_N - 2 > 0$, that is $x_N > 2$.

3. Sign chart:



4. The inequality is strict, so x_N (where the fraction equals 0) is excluded:

$$\boxed{a > 2 : \quad x < 2 \ \vee \ x > \frac{3a}{a-2}} \tag{7}$$

that is $x \in (-\infty, 2) \cup \left(\frac{3a}{a-2}, +\infty\right)$.

Memento M3 — Comparing two numbers

$A > B \iff A - B > 0$. When A and B depend on a parameter, the sign of the difference $A - B$ tells us their order on the x-axis, and that order can change as the parameter varies.

Check. With $a = 3$, (6) gives $x_N - 2 = \frac{7}{1} = 7$; indeed $x_N = \frac{9}{1} = 9$ and $9 - 2 = 7$. Inequality (1) becomes $\frac{x-9}{x-2} > 0$, which solved directly gives $x < 2 \vee x > 9$: exactly what (7) gives.

Let's recap

- For $a > 2$ the value x_N that makes the numerator zero lies to the right of 2, by (6).
- Solution: $x < 2 \vee x > \frac{3a}{a-2}$, formula (7).

4. The case $a < 2$: the second critical value

Where we are. For $a < 2$ we know that $N(x) \geq 0 \iff x \leq x_N$, formula (4), and that $D(x) \geq 0 \iff x \geq 2$, formula (2).

Goal. As before: sign chart and solution.

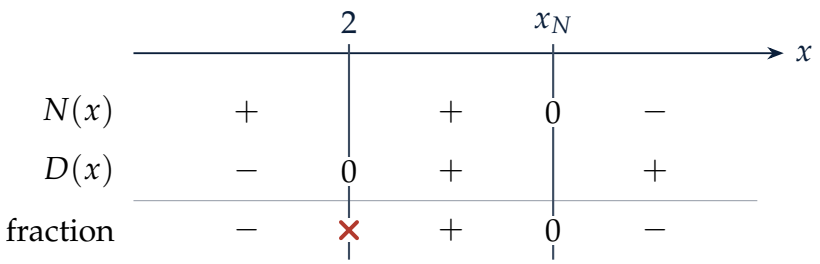
Idea. Once again we need the order of x_N and 2, given by (6). But now the denominator $a - 2$ is negative, and the sign of the difference depends on $a + 4$, which **changes sign** within this case.

1. From (6), with $a - 2 < 0$:

$$x_N - 2 > 0 \iff a + 4 < 0 \iff a < -4.$$

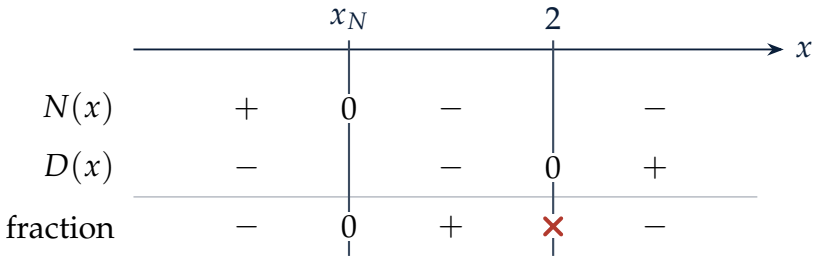
   A new value appears, $a = -4$, and we must distinguish three sub-cases.

2. **Sub-case** $a < -4$**:** $x_N > 2$.



$$a < -4 : \quad 2 < x < \frac{3a}{a-2} \tag{8}$$

3. **Sub-case $-4 < a < 2$: $x_N < 2$.**



$$\boxed{-4 < a < 2: \quad \frac{3a}{a-2} < x < 2} \tag{9}$$

4. **Sub-case $a = -4$:** here $x_N = \frac{-12}{-6} = 2$, that is, the value that makes the numerator zero falls exactly on the excluded point. We substitute $a = -4$ into (1):

$$\frac{-6x+12}{x-2} = \frac{-6(x-2)}{x-2} = -6 \qquad (x \neq 2),$$

and the inequality becomes $-6 > 0$, which is always false:

$$\boxed{a = -4: \quad \text{no solution}} \tag{10}$$

Watch out — When two points overlap

If the value that makes the numerator zero coincides with the excluded point, the sign chart "loses" a column and the fraction simplifies. Here too, the value of the parameter must be substituted into the **original** inequality.

Check.

- With $a = 0$, which belongs to the sub-case $-4 < a < 2$, (1) becomes $\frac{-2x}{x-2} > 0 \iff \frac{x}{x-2} < 0 \iff 0 < x < 2$. Formula (9) gives $x_N = 0$ and hence the same interval.
- With $a = -10$, sub-case $a < -4$, (1) becomes $\frac{-12x+30}{x-2} > 0 \iff \frac{2x-5}{x-2} < 0 \iff 2 < x < \frac{5}{2}$. Formula (8) gives $x_N = \frac{-30}{-12} = \frac{5}{2}$ and hence the same interval.

Let's recap

- **Second critical value: $a = -4$.** It arises from comparing the value that makes the numerator zero with the excluded point: for $a = -4$ the two points coincide, and crossing -4 they swap places.
- Solutions: formulas (8), (9) and (10).

5. Summary of the solution

Where we are. We have solved every case.

Goal. Collect the complete solution and check its consistency.

Values of a	Solution	Formula
$a < -4$	$2 < x < \dfrac{3a}{a-2}$	(8)
$a = -4$	no solution	(10)
$-4 < a < 2$	$\dfrac{3a}{a-2} < x < 2$	(9)
$a = 2$	$x < 2$	(5)
$a > 2$	$x < 2 \ \vee \ x > \dfrac{3a}{a-2}$	(7)

Check — consistency at the boundaries. As a approaches a critical value, the solutions of the neighbouring cases should "approach" that of the isolated case.

- **Towards $a = 2$.** As a approaches 2 from the right, $x_N = \frac{3a}{a-2}$ becomes huge and positive: the part $x > x_N$ of (7) "runs away" and $x < 2$ remains. From the left, x_N becomes huge and negative, and the interval in (9) widens to $x < 2$. Both recover (5).
- **Towards $a = -4$.** By (6), x_N approaches 2. The intervals in (8) and (9) shrink until they vanish, in agreement with (10).

Let's recap

- Two critical values, $a = -4$ and $a = 2$, split the a-axis into three intervals.
- Altogether we discuss **five cases**: the three intervals and the two isolated values.
- A figure showing the whole discussion at a glance is in Appendix A.

Part II — The answer to the question

6. Where the values to discuss come from

Where we are. We have solved the first exercise and met two critical values along the way.

Goal. Answer the initial question: how do you tell which values of the parameter to discuss?

Idea. Let us look back: neither value appeared by chance; each was forced on us by a precise **obstacle** in the solution.

- $a = 2$ appeared when we had to **divide** by $a - 2$ (section 2).
- $a = -4$ appeared when we had to **order** the points x_N and 2 on the x-axis (sections 3–4).

Generalising, the values to discuss come from three sources.

1. **Existence.** Values of a for which the inequality loses its meaning, for instance because a denominator vanishes for every x. In the first exercise this source plays no role: the domain condition, $x \neq 2$, holds for every a.
2. **Order.** Values of a for which two critical points (the values of x that make the numerator or the denominator zero) **coincide**. Crossing those values, the two points swap places and the sign chart changes. They are found by studying the sign of the difference (Memento M3). In the first exercise: $a = -4$. You may compare an expression containing the parameter with a number, as here, or two expressions that both contain the parameter.
3. **Coefficient.** Values of a for which a quantity we must divide by vanishes (and hence changes sign): faced with $f(a)\,x \geq g(a)$, we need the sign of $f(a)$. In the first exercise: $a = 2$. If $f(a)$ is more complicated, for example $f(a) = a^2 - 4$, finding its sign requires solving an inequality in a, and there may be more than one critical value, here $a = \pm 2$. The same source also applies to the denominator, when it contains the parameter.

Practical procedure.

1. **Existence:** look for the values of a for which the inequality is meaningless, and exclude them.

2. **Coefficients:** whenever you must divide by an expression in a, study its sign; mark the values where it vanishes.
3. **Order:** for every pair of critical points that depend on a, study the sign of their difference; mark the values where it vanishes.
4. Place all the values you found on the a-axis. Discuss **every interval** and, separately, **every isolated value**, substituting it into the original inequality.

Watch out — Isolated values go into the original inequality

The formulas derived along the way may lose their meaning exactly at the critical values: in the first exercise $x_N = \frac{3a}{a-2}$ does not exist for $a = 2$. That is why isolated values are handled by substituting them into the original inequality.

Let's recap

- The values to discuss are not guessed: they are the ones where the solution runs into an obstacle.
- There are three sources: **existence** (the inequality loses its meaning), **coefficient** (we don't know the sign of what we divide by), **order** (two critical points swap places).
- We discuss the intervals between the critical values and, separately, the values themselves.

Part III — The direct method

In Part I we **discovered** the critical values along the way, each time the solution got stuck. Now that we know where they come from, we can reverse the order: **first** we look for all of them with the procedure of section 6, **then** we solve.

The second exercise serves two purposes at once. It is a check of what we derived in Part II, and at the same time it shows the **direct method**, the quickest and most efficient way to tackle exercises of this kind: with the critical values already in hand, the solution holds no surprises. In a class test, this is exactly the method that lets you reach the end well, and with time to spare.

Discuss, as the parameter $a \in \mathbb{R}$ varies, the inequality

$$\frac{(a-1)x - a}{a(x-a)} \geq 0 \tag{11}$$

7. Hunting for the critical values

Where we are. Once again we have a rational inequality with a real parameter a, but this time we also have the procedure of section 6, which we will follow step by step.

Goal. Find all the values of a to discuss, before building any chart.

Idea. We go through the three sources in the order of the procedure.

1. **Existence.** For $a = 0$ the denominator equals $0 \cdot (x - 0) = 0$ for every x: (11) **loses its meaning**. The value $a = 0$ must be excluded. For $a \neq 0$ the domain condition is $x \neq a$: this time the excluded point moves with the parameter.

2. **Coefficients.**

   - Numerator: $N(x) \geq 0 \iff (a-1)x \geq a$. To divide we need the sign of $a - 1$: **critical value** $a = 1$.
   - Denominator: $D(x) = a(x - a) \geq 0$. To isolate x we must divide by a: we need the sign of a, and $a = 0$ shows up again.

   For $a \neq 1$ we call $x_N = \dfrac{a}{a-1}$ the value of x at which the numerator vanishes. We then have:

$$N(x) \geq 0 \iff \begin{cases} x \geq x_N & \text{if } a > 1 \\ x \leq x_N & \text{if } a < 1 \end{cases} \qquad D(x) \geq 0 \iff \begin{cases} x \geq a & \text{if } a > 0 \\ x \leq a & \text{if } a < 0 \end{cases} \tag{12}$$

3. **Order.** The critical points are x_N and a, and **both** depend on the parameter. We study their difference:

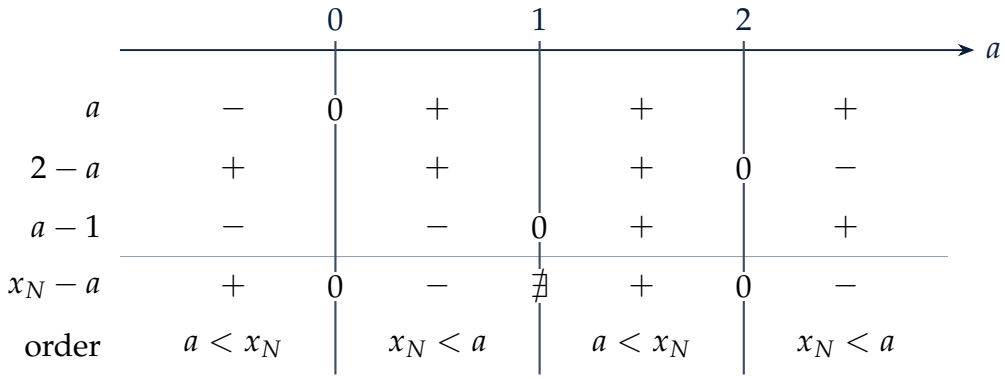
$$x_N - a = \frac{a}{a-1} - a = \frac{a - a^2 + a}{a-1},$$

   that is

$$\boxed{x_N - a = \frac{a(2-a)}{a-1}} \tag{13}$$

   The numerator vanishes for $a = 0$, already found, and for $a = 2$: **critical value** $a = 2$.

4. **Sign of** (13), interval by interval:



Let's recap

- Critical values: $a = 0$, $a = 1$, $a = 2$.
- $a = 0$ **comes from all three sources**: the inequality loses its meaning, the sign of the denominator depends on a, and x_N coincides with a. A value that turns up several times is still discussed **only once**.
- $a = 1$ comes from the coefficient of the numerator, $a = 2$ from comparing x_N with a.
- To discuss: four intervals and three isolated values, one of which ($a = 0$) is excluded.

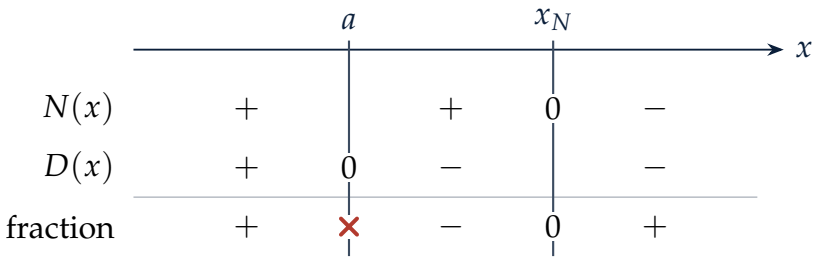
8. Solving case by case

Where we are. We know the critical values, the signs of N and D from (12), and the order of the critical points from the table in section 7.

Goal. Solve (11) in each case.

Idea. Within each interval the order of the points is fixed: all we need is the sign chart. Since the inequality is **non-strict** ($\geq$), the value x_N that makes the numerator zero is part of the solution, provided it does not coincide with the excluded point.

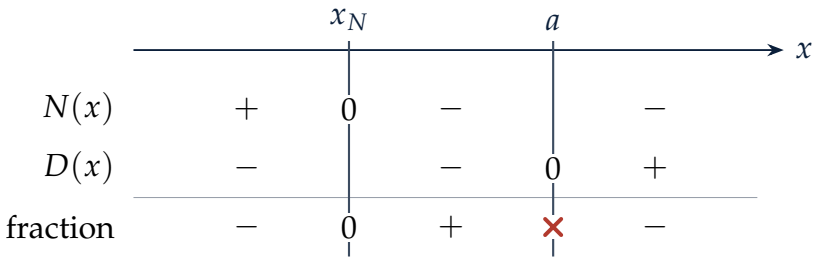
1. $a < 0$**:** $a < x_N$; $N \geq 0$ for $x \leq x_N$, $D \geq 0$ for $x \leq a$.



$$\boxed{a < 0: \quad x < a \ \vee \ x \geq \frac{a}{a-1}} \tag{14}$$

2. $a = 0$**:** the inequality loses its meaning (section 7).

3. $0 < a < 1$**:** $x_N < a$; $N \geq 0$ for $x \leq x_N$, $D \geq 0$ for $x \geq a$.



$$\boxed{0 < a < 1: \quad \frac{a}{a-1} \leq x < a} \tag{15}$$

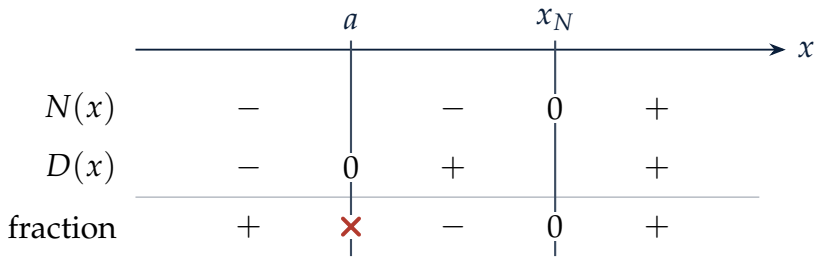
4. $a = 1$**:** substituting into (11),

$$\frac{-1}{x-1} \geq 0 \iff x - 1 < 0 \iff x < 1.$$

The fraction never vanishes, so the "equal to" adds nothing:

$$\boxed{a = 1: \quad x < 1} \tag{16}$$

5. $1 < a < 2$**:** $a < x_N$; $N \geq 0$ for $x \geq x_N$, $D \geq 0$ for $x \geq a$.



$$\boxed{1 < a < 2: \quad x < a \ \vee \ x \geq \frac{a}{a-1}} \tag{17}$$

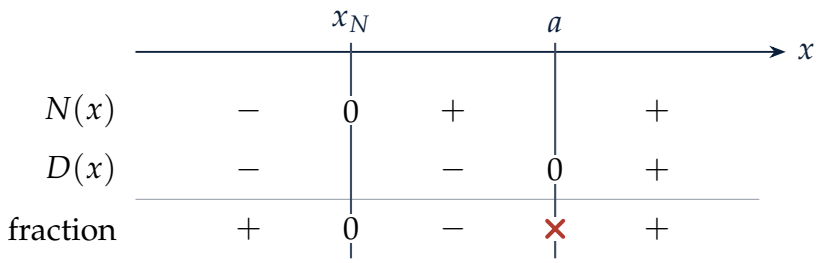
6. $a = 2$**:** here $x_N = \frac{2}{1} = 2 = a$. Substituting into (11),

$$\frac{x-2}{2(x-2)} = \frac{1}{2} \qquad (x \neq 2),$$

and $\frac{1}{2} \geq 0$ is always true:

$$\boxed{a = 2: \quad x \neq 2} \tag{18}$$

7. $a > 2$**:** $x_N < a$; $N \geq 0$ for $x \geq x_N$, $D \geq 0$ for $x \geq a$.



$$\boxed{a > 2: \quad x \leq \frac{a}{a-1} \ \vee \ x > a} \tag{19}$$

Watch out — "Equal to" and the excluded point

With the $\geq$ sign the value that makes the numerator zero is part of the solution, but **only if** it does not coincide with the point excluded by the domain conditions. In this exercise the two points coincide exactly for $a = 0$ and $a = 2$, that is, at the values found by the comparison in (13): this is no coincidence.

Let's recap

- Each interval has its own sign chart, with the order of the points given by (13).
- At the isolated values we substitute into (11): for $a = 1$ we get $x < 1$, for $a = 2$ every $x \neq 2$.

9. Summary and check

Where we are. We have solved every case.

Goal. Collect the solution and check it.

Values of a	Solution	Formula
$a < 0$	$x < a \ \vee \ x \geq \dfrac{a}{a-1}$	(14)
$a = 0$	the inequality loses its meaning	—
$0 < a < 1$	$\dfrac{a}{a-1} \leq x < a$	(15)
$a = 1$	$x < 1$	(16)
$1 < a < 2$	$x < a \ \vee \ x \geq \dfrac{a}{a-1}$	(17)
$a = 2$	$x \neq 2$	(18)
$a > 2$	$x \leq \dfrac{a}{a-1} \ \vee \ x > a$	(19)

The critical values found **before** solving, in section 7, are exactly those where the solution changes form: the procedure worked.

Check — numerical tests.

- $a = -1$. Inequality (11) becomes $\frac{-2x+1}{-(x+1)} \geq 0 \iff \frac{2x-1}{x+1} \geq 0 \iff x < -1 \vee x \geq \frac{1}{2}$. Formula (14) gives $x_N = \frac{-1}{-2} = \frac{1}{2}$: same solution.
- $a = \frac{1}{2}$. Inequality (11) becomes $\frac{-\frac{1}{2}x-\frac{1}{2}}{\frac{1}{2}\left(x-\frac{1}{2}\right)} \geq 0 \iff \frac{x+1}{x-\frac{1}{2}} \leq 0 \iff -1 \leq x < \frac{1}{2}$. Formula (15) gives $x_N = \frac{1/2}{-1/2} = -1$: same solution.
- $a = 3$. Inequality (11) becomes $\frac{2x-3}{3(x-3)} \geq 0 \iff x \leq \frac{3}{2} \vee x > 3$. Formula (19) gives $x_N = \frac{3}{2}$: same solution.

Check — consistency at the boundaries.

- **Towards $a = 1$.** From the left, in (15) x_N becomes huge and negative and the interval widens to $x < 1$. From the right, in (17) x_N becomes huge and positive and $x < 1$ remains. Both recover (16).
- **Towards $a = 2$.** In (17) and (19) both a and x_N approach 2: the two parts of the solution end up covering the whole x-axis except the point 2, in agreement with (18).
- **Towards $a = 0$.** Here something different happens. From the left, in (14) a and x_N approach 0 and the solution tends to cover **the whole x-axis**. From the right, in (15) the interval shrinks until it **vanishes**. The solution "jumps": this is the visible sign that $a = 0$ is not a critical value like the others, because there the inequality does not exist.

Let's recap

- Three critical values, $a = 0, 1, 2$, found with the procedure before solving and confirmed by the solution.
- At critical values of the "coefficient" or "order" type the solution changes continuously; at the value of "existence" type ($a = 0$) the solution jumps.

Appendix A (optional) — The whole discussion in one figure

Let us go back to the first exercise:

$$\frac{(a-2)x-3a}{x-2} > 0 \tag{1}$$

So far we have fixed a and looked for the values of x. Let us instead look at **all the pairs** (a, x) together, in a plane with a on the horizontal axis and x on the vertical one.

The two key lines.

- The excluded point, $x = 2$, is a **horizontal line**.
- The value that makes the numerator zero, $x = \frac{3a}{a-2}$, can be rewritten as

  $$x = 3 + \frac{6}{a-2},$$

  which is a **hyperbola** with vertical asymptote $a = 2$ and horizontal asymptote $x = 3$.

The numerator can also be rewritten so that the hyperbola becomes recognisable:

$$N = (a-2)x - 3a = (a-2)(x-3) - 6.$$

How the blue region is built. We simply transfer to the plane the solution found in section 5, interval by interval. For each value of a the solutions are one or more segments of the corresponding vertical line: colouring these segments for all the values of a in an interval gives a region of the plane.

- For $a < -4$, from (8): $2 < x < \frac{3a}{a-2}$, that is, the points **between the line** $x = 2$ **and the hyperbola**, which here lies above the line.
- For $-4 < a < 2$, from (9): $\frac{3a}{a-2} < x < 2$, that is, the points **between the hyperbola**, which here lies below the line, **and the line** $x = 2$.
- For $a > 2$, from (7): $x < 2$ or $x > \frac{3a}{a-2}$, that is, **two regions**: everything below the line $x = 2$ and everything above the right branch of the hyperbola.
- For $a = -4$, (10) gives no solutions. For $a = 2$, (5) gives the vertical half-line $x < 2$, which has no area and therefore does not show up as a coloured region.

The boundaries are not part of the region: on the hyperbola the fraction equals 0, while (1) requires it to be strictly positive; on the line $x = 2$ the fraction does not exist.

How to read the figure. Fixing a value of a means drawing a **vertical line**. The points of that line falling inside the shaded region are the solutions of (1) for that value of a.

Where the critical values are. They are exactly the values of a where the figure "changes":

- $a = -4$ is the abscissa of the point where the left branch of the hyperbola (the one with $a < 2$) **meets** the line $x = 2$: before -4 the branch lies above the line, after it below. The right branch ($a > 2$) instead always stays above, since there $x > 3$. It is the critical value of "order" type.
- $a = 2$ is the **vertical asymptote**: there the hyperbola does not exist, just like x_N. It is the critical value of "coefficient" type.

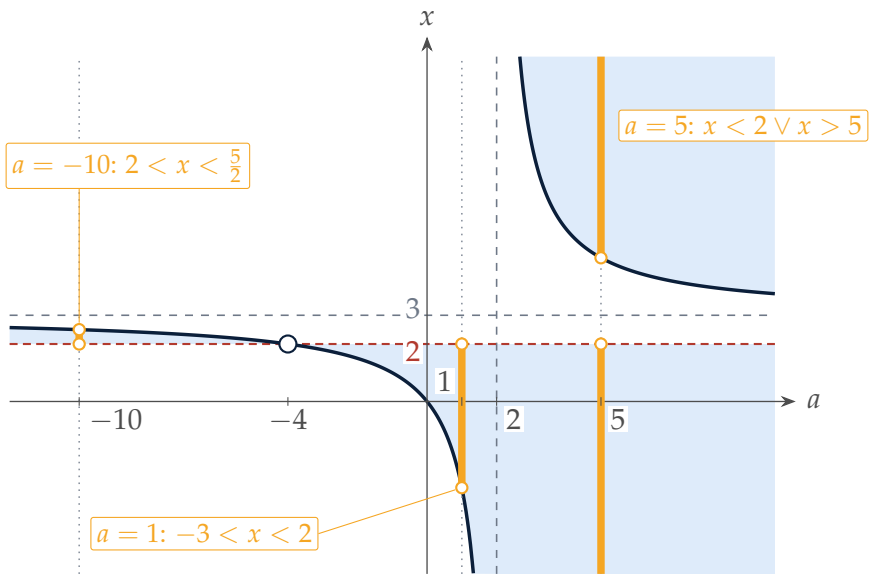


Figure A.1. The solutions of (1) in the (a, x) plane. For a fixed a, the solutions are the points of the vertical line that fall inside the blue region. In orange, three examples, $a = -10$, $a = 1$ and $a = 5$, with the solution written beside them; the open circles mark the excluded endpoints.

Let's recap

- In the (a, x) plane the discussion becomes a figure: a region bounded by a line and a hyperbola.
- The critical values are the abscissas where the figure changes: an intersection ($a = -4$) and an asymptote ($a = 2$).

About the author. Nicola Maglione is an electronic engineer (degree in Electronic Engineering, University of Naples, 2001), with over twenty years of experience in the semiconductor industry as a design and program manager, specialized in NAND Flash memory and high-speed interfaces, and holder of several patents in memory architectures. Alongside his professional work, he has always cultivated a passion for mathematics — particularly Analysis, Abstract Algebra, and Number Theory — and for physics, which he shares through keenonscience, his science-outreach page, and on the website keenonmaths.org.

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